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Calculus I Problem Sheets: Complex numbers

(9a) Write $\sqrt{-4}$ as a complex number.

$\sqrt{-4} = \sqrt{-1}\sqrt{4} = i2 = 2i = 0 + 2i.$

(9b) Write $\sqrt{-6}$ as a complex number

$\sqrt{-6} = \sqrt{-1}\sqrt{6} = i\sqrt{6} = \sqrt{6}i.$

(9c) Write $2\sqrt{-12}$ as a complex number

$2\sqrt{-12} = 2 \cdot 2\sqrt{-1}\sqrt{3} = 4\sqrt{3}i.$

(10a) Find $Re(z)$ and $Im(z)$ when $z = 3-4i.$

$Re(z) = Re(3-4i) = 3$ and $Im(z) = Im(3-4i) = -4.$

(10b) Find $Re(z)$ and $Im(z)$ when $z = 6i.$

$Re(z) = Re(6i) = Re(0+6i) = 0$ and $Im(z) = Im(6i) = 6.$

(10c) Find $Re(z)$ and $Im(z)$ when $z = 2+7i$

$Re(z) = Re(2+7i) = 2$ and $Im(z) = Im(2+7i) = 7.$

(10d) Find $Re(z)$ and $Im(z)$ when $z = 5/7.$

$Re(z) = Re(5/7) = Re(5/7 + 0i) = 5/7$ and $Im(z) = Im(5/7 + 0i) = 0.$

(10e) Find $Re(z)$ and $Im(z)$ when $z = \frac{1}{2}i + 2.$

$Re(z) = Re(2 + \frac{1}{2}i) = 2$ and $Im(z) = Im(2 + \frac{1}{2}i) = \frac{1}{2}.$

(11) Let $z_1 = 2+6i$, $z_2 = \frac{1}{2}+2i$, $z_3 = -6i$, $z_4 = -3+2i$, $z_5 = 5$.

(11a) Simplify $z_1 + z_4$.

$z_1 + z_4 = (2+6i) + (-3+2i) = -1+8i$.

(11b) Simplify $z_1 + z_3$.

$z_1 + z_3 = (2+6i) + (-6i) = 2+6i-6i = 2$.

(11c) Simplify $z_1 - 2z_2$

$z_1 - 2z_2 = (2+6i) - 2(\frac{1}{2}+2i) = 2+6i-1-4i = 1-2i$.

(11d) Simplify $z_4 + z_3$.

$z_4 + z_3 = -3+2i+(-6i) = -3+2i-6i = -3-4i$.

(11e) Simplify $z_2 + z_4$

$z_2 + z_4 = \frac{1}{2}+2i+(-3+2i) = \frac{1}{2}-3+2i+2i = -\frac{5}{2}+4i$.

(11f) Simplify $3z_2 - 2z_4$.

$3z_2 - 2z_4 = 3(\frac{1}{2}+2i) - 2(-3+2i) = \frac{3}{2}+6i+6-4i = \frac{15}{2}+2i$.

(11g) Simplify $z_5 + z_3$

$z_5 + z_3 = 5 + (-6i) = 5-6i$.

(11h) Simplify $z_3 + z_2$

$z_3 + z_2 = (-6i) + \frac{1}{2}+2i = \frac{1}{2}-4i$.

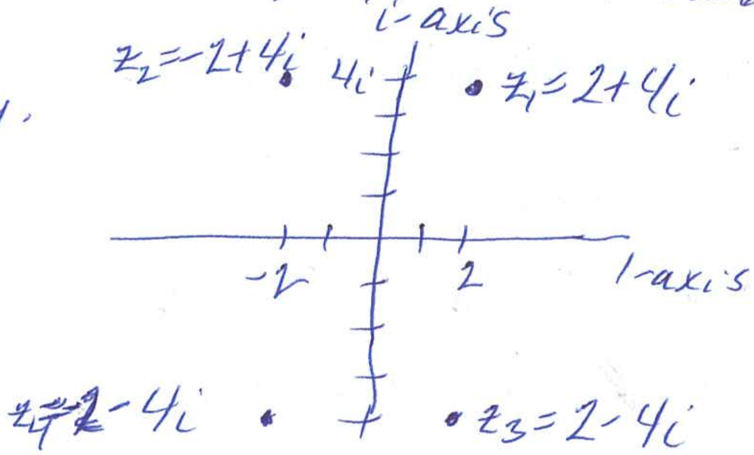
(11i) Simplify $2z_1 + z_2 + 3z_3$

$2z_1 + z_2 + 3z_3 = 2(2+6i) + \frac{1}{2}+2i+3(-6i) = 4+12i+\frac{1}{2}+2i-18i$

$= \frac{9}{2} + (14-18)i = \frac{9}{2} - 4i$.

(12) Let $z_1 = 2+4i$, $z_2 = -2+4i$, $z_3 = 2-4i$, $z_4 = -2-4i$.

(12a) Graph z_1, z_2, z_3, z_4.

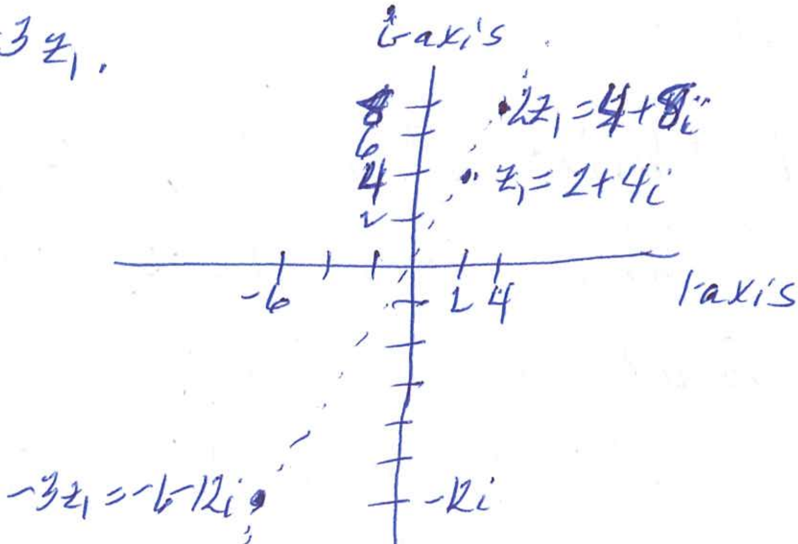


(12b) Graph z_1, $2z_1$ and $-3z_1$.

$z_1 = 2+4i$,

$2z_1 = 2(2+4i) = 4+8i$;

$-3z_1 = -3(2+4i) = -6-12i$



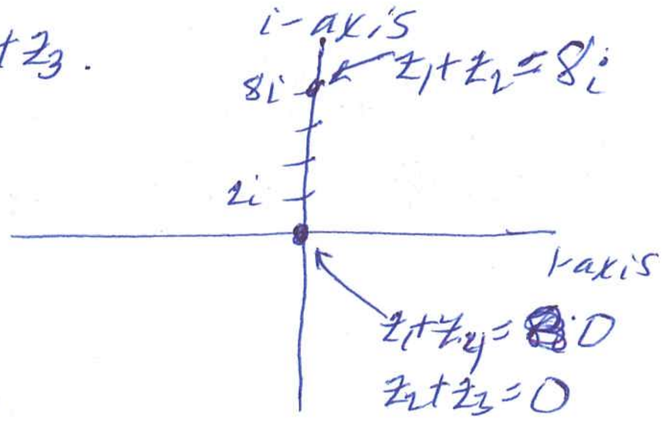
All the points lie on a line through the origin.

(12c) Graph z_1+z_2, z_1+z_4 and z_2+z_3.

$z_1+z_2 = (2+4i)+(-2+4i) = 8i$

$z_1+z_4 = (2+4i)+(-2-4i) = 0$

$z_2+z_3 = (-2+4i)+(2-4i) = 0$



(12di) Is it true that $z_1 = -z_2$?

$-z_2 = -(-2+4i) = 2-4i$ and $z_1 = 2+4i$.

So $z_1 \neq -z_2$.

(12dii) Is it true that $z_1 = -z_3$.

$-z_3 = -(2-4i) = -2+4i$ and $z_1 = 2+4i$

So $-z_3 \neq z_1$

(12diii) Is it true that $z_1 = -z_4$?

$-z_4 = -(-2-4i) = 2+4i$ and $z_1 = 2+4i$

So $-z_4 = z_1$

(12div) Is it true that $z_2 = -z_3$.

$-z_3 = -(2-4i) = -2+4i$ and $z_2 = -2+4i$.

So $z_2 = -z_3$.

(13) Write i^2, i^3, i^4 in Cartesian form.

Write $i^{23} = (i^4)^5 \cdot i^3$, i^{14} and i^{259} in Cartesian form

Solution:

$i^2 = -1, \quad i^3 = (-1)i = -i, \quad i^4 = i^2 \cdot i^2 = (-1)(-1) = 1.$

$i^{23} = (i^4)^5 \cdot i^3 = i^3 = -i$

$i^{14} = (i^4)^3 i^2 = 1 \cdot (-1) = -1.$

$i^{259} = i^{260-1} = i^{260} i^{-1} = i^{-1} = -i$