Calculus 2 Lect. 17 04.09.2019

R. Ram (1)

Example 5.10 Solve $\frac{dy}{dx} = \frac{y}{x} + \cos^2\left(\frac{y}{x}\right)$

by substituting $u = \frac{y}{x}$.

Solution: Let $u = y/x$. Then $u = yx^{-1}$ and $y = ux$

and $\frac{dy}{dx} = u + \frac{du}{dx}x$. So

$$u + \frac{du}{dx}x = \frac{ux}{x} + \cos^2\left(\frac{ux}{x}\right) = u + \cos^2 u.$$

So $$\frac{du}{dx}x = \cos^2 u.$$

So $$\frac{1}{\cos^2 u}\frac{du}{dx} = \frac{1}{x}.$$

So $$\int \sec^2 u \frac{du}{dx}\,dx = \int \frac{1}{x}\,dx$$

So $\tan u = \log x + c$, where c is a constant

So $u = \arctan(\log x + c)$

So $\frac{y}{x} = \arctan(\log x + c)$.

So $y = x \arctan(\log x + c)$ //.

Example 5.11 Solve $\frac{dy}{dx} + y = e^{3x} y^4$ by substituting $u = y^{-3}$.

Solution: Let $u = y^{-3}$. Then $y = u^{-\frac{1}{3}}$ and

$$\frac{dy}{dx} = -\frac{1}{3} u^{-4/3} \frac{du}{dx}.$$

Since $\frac{dy}{dx} + y = e^{3x} y^4$, then

$$-\frac{1}{3} u^{-4/3} \frac{du}{dx} + u^{-\frac{1}{3}} = e^{3x} (u^{-\frac{1}{3}})^4 = e^{3x} e^{-4/3}$$

So $\frac{du}{dx} - 3u = -3e^{3x}$ (by multiplying both sides by $-3u^{4/3}$).

So $e^{-3x} \frac{du}{dx} - 3e^{-3x} u = -3$ (by multiplying both sides by e^{-3x}).

So $$\frac{d(e^{-3x}u)}{dx} = -3$$

So $\int \frac{d(e^{-3x}u)}{dx} dx = \int -3\, dx = -3x + c.$

So $e^{-3x} u = (-3x + c)$ and $u = (-3x + c) e^{3x}$.

So $y^{-3} = (-3x + c) e^{3x}$

So $y = (-3x + c)^{-\frac{1}{3}} e^{-x}$, where c is a constant. //