Calculus 2 Lecture 30

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Example 7.11 Let $z = f(x,y) = x^2 + 3xy - y^2$.

If x changes from 2 to 2.05 and y changes from 3 to 2.96 estimate the change in z

Solution $\Delta x = 2.05 - 2 = 0.05$

$\Delta y = 2.96 - 3 = -0.04$

Then

$$\Delta z = \Delta f = \left.\frac{\partial f}{\partial x}\right]_{(x,y)=(2,3)} \Delta x + \left.\frac{\partial f}{\partial y}\right]_{(x,y)=(2,3)} \Delta y$$

and

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2+3xy-y^2) = 2x+3y-0 = 2x+3y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2+3xy-y^2) = 0+3x-2y = 3x-2y.$$

So $\left.\frac{\partial f}{\partial x}\right]_{(x,y)=(2,3)} = 2\cdot 2 + 3\cdot 3 = 4+9 = 13$ and

$$\left.\frac{\partial f}{\partial y}\right]_{(x,y)=(2,3)} = 3\cdot 2 - 2\cdot 3 = 6-6 = 0.$$

So $\Delta f = \left.\frac{\partial f}{\partial x}\right]_{(x,y)=(2,3)} \Delta x + \left.\frac{\partial f}{\partial y}\right]_{(x,y)=(2,3)} \Delta y$

$$= 13\cdot 0.05 + 0\cdot(-0.04) = 0.65 - 0 = 0.65$$

<u>Example 7.12</u> Find the linear approximation of $f(x,y) = xe^{xy}$ at $(1,0)$. Hence approximate $f(1.1, -0.1)$.

<u>Solution</u>: The linear approximation to f at $(1,0)$ is

$$f(x,y) \approx f(1,0) + \left.\frac{\partial f}{\partial x}\right]_{(x,y)=(1,0)} (x-1) + \left.\frac{\partial f}{\partial y}\right]_{(x,y)=(1,0)} (y-0)$$

Now,

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(xe^{xy}) = xe^{xy}\cdot y + 1\cdot e^{xy} = xye^{xy} + e^{xy}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(xe^{xy}) = x\cdot e^{xy}\cdot x = x^2 e^{xy}.$$

So

$$\left.\frac{\partial f}{\partial x}\right]_{(x,y)=(1,0)} = \left. xye^{xy} + e^{xy}\right]_{(x,y)=(1,0)} = 1\cdot 0 e^{1\cdot 0} + e^{1\cdot 0}$$

$$= 0 + e^0 = 1 \quad \text{and}$$

$$\left.\frac{\partial f}{\partial y}\right]_{(x,y)=(1,0)} = \left. x^2 e^{xy}\right]_{(x,y)=(1,0)} = 1^2 e^{1\cdot 0} = 1\cdot e^0 = 1.$$

So

$$f(x,y) \approx f(1,0) + \left.\frac{\partial f}{\partial x}\right]_{(x,y)=(1,0)} (x-1) + \left.\frac{\partial f}{\partial y}\right]_{(x,y)=(1,0)} (y-0)$$

$$= 1\cdot e^{1\cdot 0} + 1\cdot(x-1) + 1\cdot(y-0)$$

$$= 1 + x - 1 + y = x + y.$$

So

$$f(1.1, -0.1) \approx 1.1 + (-0.1) = 1. \quad //.$$