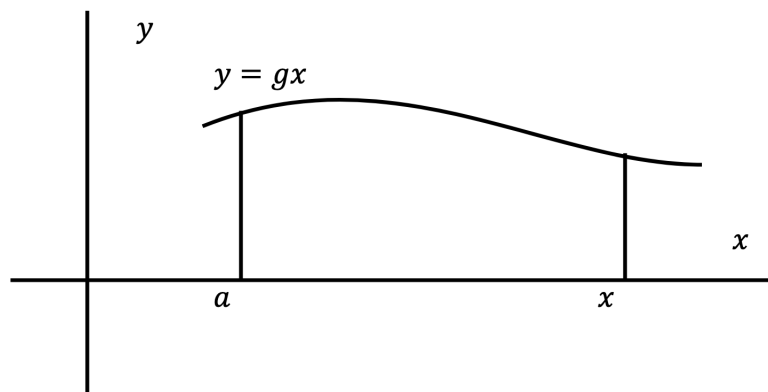## 1.25 Why the fundamental theorem of calculus works

The fundamental theorem of calculus says that
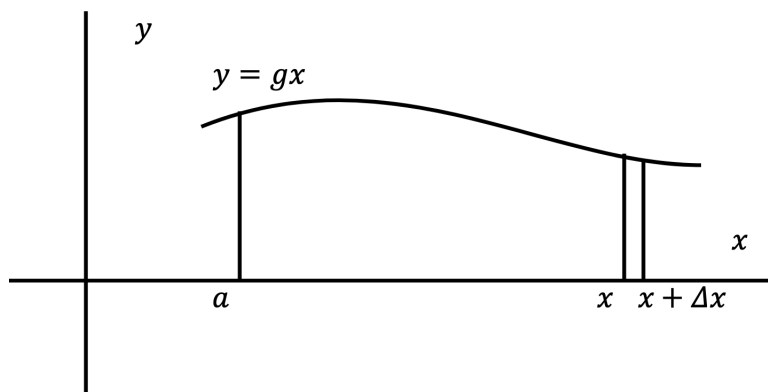
$$\text{(Area under } g(x) \text{ from } a \text{ to } b) = A(b) - A(a), \qquad \text{where} \quad \int g(x)dx = A(x) + c.$$

Why does this work?

Let $A(x) = $ (area under $g(x)$ from a to x).

Area under $g(x)$ from a to b

Difference in area is the last little box

Then

$$\begin{aligned}
\frac{dA}{dx} &= \lim_{\Delta x \to 0} \frac{A(x) - \Delta x - A(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\text{area of last little box}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{g(x)\Delta x}{\Delta x} \\
&= g(x).
\end{aligned}$$

So

$$\begin{aligned}
A(b) - A(a) &= \text{(area under } g(x) \text{ from } a \text{ to } b) - \text{(area under } g(x) \text{ from } a \text{ to } a) \\
&= \text{(area under } g(x) \text{ from } a \text{ to } b).
\end{aligned}$$