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Assignment 1 Question 4F

(a) Work in ℓ^∞ so that the norm is $\| \|_\infty$.

$$\hat{c}_c = \left\{ \lim_{n\to\infty} w_n \;\middle|\; (w_1, w_2, \ldots) \text{ is a Cauchy sequence in } c_c \right\}$$

$$\overline{c_c} = \left\{ z \in \ell^\infty \;\middle|\; \text{there is a sequence } (w_1, w_2, \ldots) \text{ in } c_c \text{ with } \lim_{n\to\infty} w_n = z \right\}.$$

If $(w_1, w_2, \ldots)$ is a sequence in c_c with $\lim_{n\to\infty} w_n = z$ then $(w_1, w_2, \ldots)$ is a Cauchy sequence in c_c and so $z \in \hat{c}_c$.

So $\overline{c_c} \subseteq \hat{c}_c$.

By Question 4B parts (a) and (d), $\overline{c_c} = c_0$.

So $c_0 \subseteq \hat{c}_c$.

If $z \in \hat{c}_c$ and $(w_1, w_2, \ldots)$ is a Cauchy sequence in c_c with $\lim_{n\to\infty} w_n = z$ then $z \in \overline{c_c}$.

So $\hat{c}_c \subseteq \overline{c_c} = c_0$.

So $\hat{c}_c = c_0$ in ℓ^∞.

(b) The same argument as for part (a) applies to get $\hat{c}_c = \overline{c_c}$ in ℓ^p, and by part (b) of Question 4B, $\overline{c_c} = \ell^p$. So $\hat{c}_c = \ell^p$ in ℓ^p.