Question 4

Let $(X, \mathcal{T})$ be a topological space. Let $S \subseteq X$.

(a) The set S is connected if there do not exist open sets U and V of X such that

$S \cap U \neq \emptyset$, $S \cap V \neq \emptyset$, $S \subseteq U \cup V$ and

$(S \cap U) \cap (S \cap V) = \emptyset$.

(b) The set S is path connected if $\underbrace{\text{}}_{S \text{ satisfies: If } x, y \in S,}$ there exists a continuous function

$p : \mathbb{R}_{[0,1]} \to S$ such that $p(0) = x$ and

$p(1) = y$.

(c) Assume S is path connected.

To show: S is connected.

To show: If S is not connected then S is not path connected

Assume S is not connected.

Let U, V be open sets in X such that

$S \cap U \neq \emptyset$, $S \cap V \neq \emptyset$, $S \subseteq U \cup V$ and

$(S \cap U) \cap (S \cap V) = \emptyset$.

Let $x \in S \cap U$ and $y \in S \cap V$ MHSAss3 Q4 2

Let $p : \mathbb{R}_{[0,1]} \to S$ with $p(0) = x$ and $p(1) = y$.

To show: p is not continuous.

Let $A = p^{-1}(S \cap U)$ and $B = p^{-1}(S \cap V)$

Then $A \cup B = \mathbb{R}_{[0,1]}$ and $A \cap B = \phi$

and $x \in A$ and $y \in B$.

~~Ed~~ Since $\mathbb{R}_{[0,1]}$ is connected then

A is not open in $\mathbb{R}_{[0,1]}$ or

B is not open in $\mathbb{R}_{[0,1]}$.

So p is not continuous.

So p is not path connected. //.