

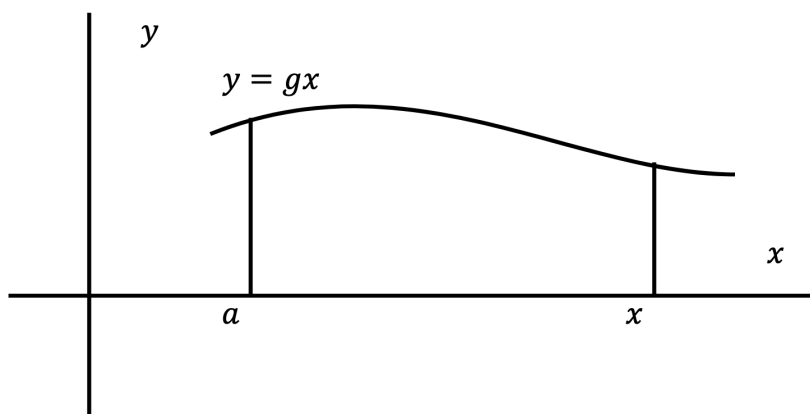
### 1.25 Why the fundamental theorem of calculus works

The fundamental theorem of calculus says that

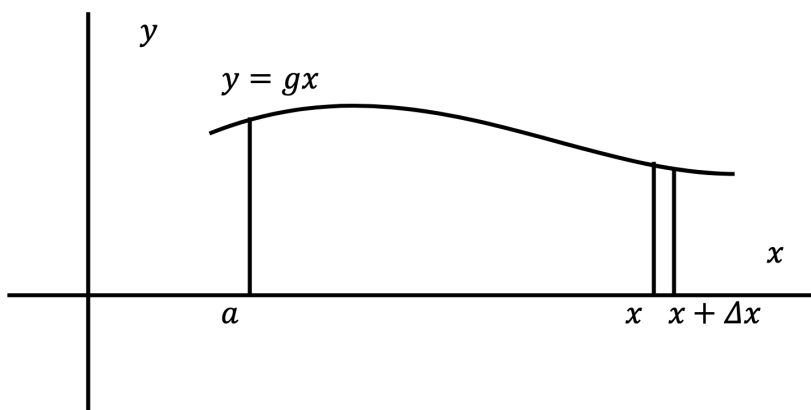
$$(\text{Area under } g(x) \text{ from } a \text{ to } b) = A(b) - A(a), \quad \text{where } \int g(x)dx = A(x) + c.$$

Why does this work?

Let  $A(x) = (\text{area under } g(x) \text{ from } a \text{ to } x)$ .



Area under  $g(x)$  from  $a$  to  $b$



Difference in area is the last little box

Then

$$\begin{aligned} \frac{dA}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{A(x + \Delta x) - A(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\text{area of last little box}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{g(x)\Delta x}{\Delta x} \\ &= g(x). \end{aligned}$$

So

$$\begin{aligned} A(b) - A(a) &= (\text{area under } g(x) \text{ from } a \text{ to } b) - (\text{area under } g(x) \text{ from } a \text{ to } a) \\ &= (\text{area under } g(x) \text{ from } a \text{ to } b). \end{aligned}$$