6.3 Diagonalization

Theorem 6.3.

(a) Let $A \in M_n(\mathbb{F})$. The matrix A has n linearly independent eigenvectors $p_1, \ldots, p_n \in \mathbb{F}^n$ with eigenvalues $\lambda_1, \ldots, \lambda_n$ if and only if $A = PDP^{-1}$ where,

$$P = \begin{pmatrix} | & & | \\ p_1 & \cdots & p_n \\ | & & | \end{pmatrix} \qquad \text{and} \qquad D = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

so that $p_1, \ldots, p_n$ are the columns of P and D is the diagonal matrix with diagonal entries $\lambda_1, \ldots, \lambda_n$.

(b) Assume that the inner product on $\mathbb{F}^n$ is given by (stdSinnprod). Let $A \in M_n(\mathbb{F})$. The matrix A has n orthonormal eigenvectors $q_1, \ldots, q_n \in \mathbb{F}^n$ with eigenvalues $\lambda_1, \ldots, \lambda_n$ if and only if $A = QDQ^{-1}$ and $QQ^t = 1$ where,

$$Q = \begin{pmatrix} | & & | \\ q_1 & \cdots & q_n \\ | & & | \end{pmatrix} \qquad \text{and} \qquad D = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

so that $u_1, \ldots, u_n$ are the columns of Q and D is the diagonal matrix with diagonal entries $\lambda_1, \ldots, \lambda_n$.

(c) Assume that the inner product on $\mathbb{F}^n$ is given by (stdHinnprod). Let $A \in M_n(\mathbb{F})$. The matrix A has n orthonormal eigenvectors $u_1, \ldots, u_n \in \mathbb{F}^n$ with eigenvalues $\lambda_1, \ldots, \lambda_n$ if and only if $A = UDU^{-1}$ and $U\overline{U}^t = 1$ where,

$$U = \begin{pmatrix} | & & | \\ u_1 & \cdots & u_n \\ | & & | \end{pmatrix} \qquad \text{and} \qquad D = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{pmatrix}$$

so that $u_1, \ldots, u_n$ are the columns of U and D is the diagonal matrix with diagonal entries $\lambda_1, \ldots, \lambda_n$.