

**Topic 2. Example 14.** The (2,3) cofactor of  $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$  is

$$(-1)^{2+3} \cdot \det \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = (-1)^5 \cdot (1 - 0) = -1.$$

**Topic 2. Example 15.** Using coset expansion for the third row,

$$\begin{aligned} \det \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix} &= (-1)^{3+1} \cdot 0 \cdot \det \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} + (-1)^{3+2} \cdot 1 \cdot \det \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} + (-1)^{3+3} \cdot 3 \cdot \det \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \\ &= 0 + (-1) \cdot (1 \cdot 1 - (-1) \cdot 1) + 3 \cdot (1 \cdot 1 - (-1) \cdot 2) = -2 + 9 = 7. \end{aligned}$$

**Topic 2. Example 16.** Using coset expansion for the fourth column,

$$\begin{aligned} \det \begin{pmatrix} 1 & -2 & 0 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -4 & 2 & 4 \end{pmatrix} &= (-1)^{1+4} \cdot 1 \cdot \det \begin{pmatrix} 3 & 2 & 2 \\ 1 & 0 & 1 \\ 0 & -4 & 2 \end{pmatrix} + 0 + 0 + (-1)^{4+4} \cdot 4 \cdot \det \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix} \\ &= - \left( (-1)^{1+1} \cdot 3 \cdot \det \begin{pmatrix} 0 & 1 \\ -4 & 2 \end{pmatrix} + (-1)^{2+1} \cdot 1 \cdot \det \begin{pmatrix} 2 & 2 \\ -4 & 2 \end{pmatrix} + 0 \right) \\ &\quad + 4 \left( (-1)^{1+1} \cdot 1 \cdot \det \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix} + (-1)^{1+2} \cdot (-2) \cdot \det \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} + 0 \right) \\ &= -3(0 + 4) + (4 + 8) + 4((2 - 0) + 2(2 - 3)) = -12 + 12 + 8 + 8 = 16. \end{aligned}$$