Let G be a graph with vertices labeled $1, 2, \ldots, n$.

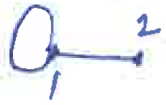
The adjacency matrix of G is the matrix $A \in M_{n\times n}(\mathbb{Q})$ with

$$A(i,j) = \begin{cases} 1, & \text{if } (i,j) \text{ is an edge in } G \\ 0, & \text{if } (i,j) \text{ is not and edge in } G \end{cases}$$

Example

G  has $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

and

$G =$  has $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

Theorem Let G be a graph and let A be the adjacency matrix of G. Let $k \in \mathbb{Z}_{>0}$. Then the (i,j) entry of A^k is

$$A^k(i,j) = \begin{pmatrix} \#\text{ of paths of length } k \\ \text{from } i \text{ to } j \text{ in } G \end{pmatrix}$$

Examples All of the following have $\det(A-t)=(t-3)^4$

$$A=\begin{pmatrix}3&&&\\&3&&\\&&3&\\&&&3\end{pmatrix}\qquad \ker(A-3)=\mathrm{span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\}$$

$$A=\begin{pmatrix}3&1&&\\&3&&\\&&3&1\\&&&3\end{pmatrix}\qquad \ker(A-3)=\mathrm{span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\right\}$$

$$A=\begin{pmatrix}3&1&&\\&3&1&\\&&3&1\\&&&3\end{pmatrix}\qquad \ker(A-3)=\mathrm{span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\right\}$$

$$A=\begin{pmatrix}3&&&\\&3&1&\\&&3&1\\&&&3\end{pmatrix}\qquad \ker(A-3)=\mathrm{span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\right\}$$

$$A=\begin{pmatrix}3&&&\\&3&1&\\&&3&\\&&&3\end{pmatrix}\qquad \ker(A-3)=\mathrm{span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\}$$

$$A=\begin{pmatrix}3&1&&\\&3&&\\&&3&\\&&&3\end{pmatrix}\qquad \ker(A-3)=\mathrm{span}\left\{\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\}$$