

### 1.5 Circular functions

Let  $i \in \mathbb{C}$  be such that  $i^2 = -1$ . The *circular functions*  $\sin z$  and  $\cos z$  are given by

$$\begin{aligned} \sin z &= \frac{e^{iz} - e^{-iz}}{2i} = z - \frac{1}{3!}z^3 + \frac{1}{5!}z^5 - \dots \quad \text{and} \\ \cos z &= \frac{e^{iz} + e^{-iz}}{2} = 1 - \frac{1}{2!}z^2 + \frac{1}{4!}z^4 - \dots, \end{aligned}$$

so that

$$e^{iz} = \cos z + i \sin z.$$

For  $a, \theta \in \mathbb{R}$ ,

$$e^{a+i\theta} = e^a e^{i\theta} = r e^{i\theta} = r(\cos \theta + i \sin \theta) = (r \cos \theta) + i(r \sin \theta) = x + iy,$$

are frequently (but not universally) used notational transformations. Define

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{1}{\tan x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}.$$

### 1.6 Hyperbolic functions

The *hyperbolic functions*  $\sinh z$  and  $\cosh z$  are given by

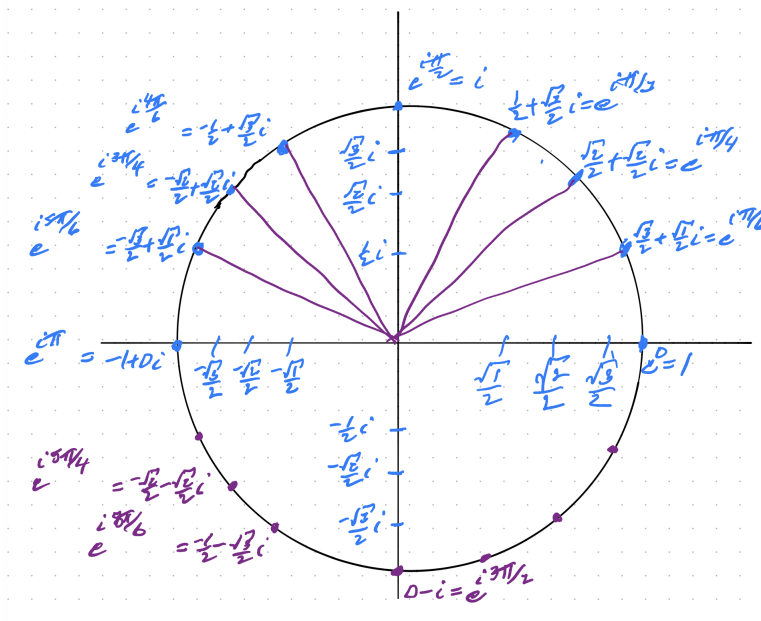
$$\begin{aligned} \sinh z &= \frac{e^z - e^{-z}}{2} = z + \frac{1}{3!}z^3 + \frac{1}{5!}z^5 + \dots \quad \text{and} \\ \cosh z &= \frac{e^z + e^{-z}}{2} = 1 + \frac{1}{2!}z^2 + \frac{1}{4!}z^4 + \dots, \end{aligned}$$

so that

$$e^z = \cosh z + \sinh z.$$

Define

$$\tanh x = \frac{\sinh x}{\cosh x}, \quad \coth x = \frac{1}{\tanh x}, \quad \operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\sinh x}.$$



sines and cosines of the favorite angles