

1 Single pages

1.1 Numbers and intervals

The positive integers: $\mathbb{Z}_{>0} = \{1, 2, 3, \dots\}$.

The nonnegative integers: $\mathbb{Z}_{\geq 0} = \{0, 1, 2, 3, \dots\}$.

The integers: $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

The rational numbers: $\mathbb{Q} = \left\{ \frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}_{\neq 0} \text{ and } \frac{a}{b} = \frac{c}{d} \text{ if } ad = bc \right\}$.

The real numbers:

$$\mathbb{R} = \{ \pm a_\ell a_{\ell-1} \dots a_1 a_0 . a_{-1} a_{-2} \dots \mid \ell \in \mathbb{Z}_{\geq 0}, a_i \in \{0, \dots, 9\}, a_\ell \neq 0 \text{ if } \ell > 0 \}.$$

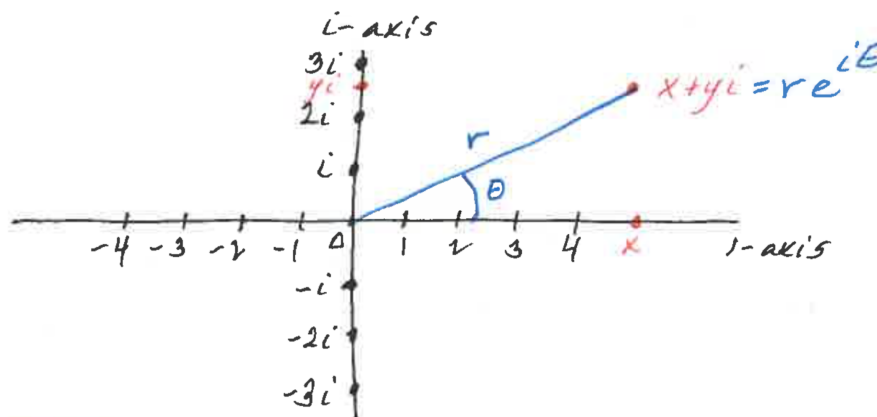
with a convention that if $a_k \neq 9$ then $\pm a_\ell \dots a_{k+1} a_k 9999 \dots = \pm a_\ell \dots a_{k+1} (a_k + 1) 000 \dots$
 so that, for example, $0.9999 \dots = 1.0000 \dots$

The complex numbers:

$$\mathbb{C} = \{ a + bi \mid a, b \in \mathbb{R} \} \quad \text{with } i^2 = -1.$$

Let $a, b \in \mathbb{R}$ with $a < b$. Define

$$\begin{aligned} \mathbb{R}_{(a,b)} &= \{ x \in \mathbb{R} \mid a < x < b \}, & \mathbb{R}_{[a,b)} &= \{ x \in \mathbb{R} \mid a \leq x < b \} \\ \mathbb{R}_{(a,b]} &= \{ x \in \mathbb{R} \mid a < x \leq b \}, & \mathbb{R}_{[a,b]} &= \{ x \in \mathbb{R} \mid a \leq x \leq b \} \\ \mathbb{R}_{(a,\infty)} &= \{ x \in \mathbb{R} \mid a < x \}, & \mathbb{R}_{[a,\infty)} &= \{ x \in \mathbb{R} \mid a \leq x \} \\ \mathbb{R}_{(-\infty,a)} &= \{ x \in \mathbb{R} \mid x < a \}, & \mathbb{R}_{(-\infty,a]} &= \{ x \in \mathbb{R} \mid x \leq a \}. \end{aligned}$$



Picture of $\mathbb{Z} \subseteq \mathbb{R} \subseteq \mathbb{C}$