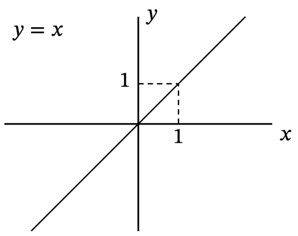
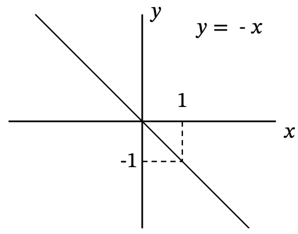
6.6 basic graphs

6.6.1 The basic line $y = x$

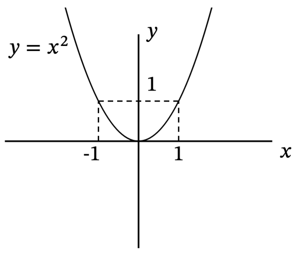


The line $y = x$

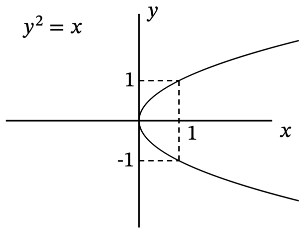


the line $y = -x$

6.6.2 The basic parabola $y = x^2$

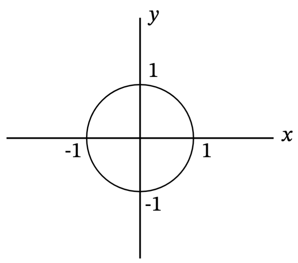


The parabola $y = x^2$



the parabola $y^2 = -x$

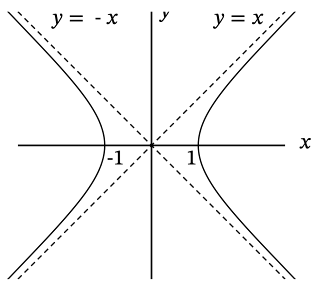
6.6.3 The basic circle $x^2 + y^2 = 1$



The circle $x^2 + y^2 = 1$

All points that are distance 1 from the origin.

6.6.4 The basic hyperbola $x^2 - y^2 = 1$



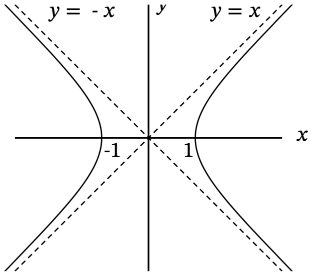
The hyperbola $x^2 - y^2 = 1$

See Section 6.7 for notes on how to derive the graph of the basic hyperbola $x^2 - y^2 = 1$.

6.7 Asymptotes

A **asymptote** of a graph $y = f(x)$ as $x \to a$ is another graph $y = g(x)$ that the original graph $y = f(x)$ gets closer and closer to as x gets closer and closer to a.

Example: Graph the basic hyperbola $x^2 - y^2 = 1$.



The hyperbola $x^2 - y^2 = 1$

Graphing notes:

(a) If $y = 0$ then $x^2 = 1$. So $x = \pm 1$.

(b) If $x = 0$ then $-y^2 = 1$ which is impossible for $y \in \mathbb{R}$.

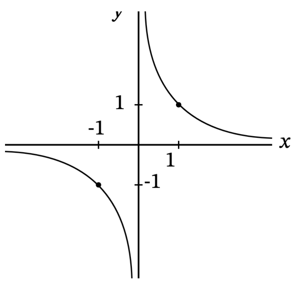
(c) The equation is $1 - \left(\frac{y}{x}\right)^2 = \left(\frac{1}{x}\right)^2$.

If x gets very big then $\frac{1}{x}$ gets closer and closer to 0 and the equation gets closer and closer to $1 - \left(\frac{y}{x}\right)^2 = 0$. This is the same as $\left(\frac{y}{x}\right)^2 = 1$, which is the same as $\frac{y}{x} = \pm 1$, i.e. $y = \pm x$. So, as x gets very large the equation gets closer and closer to $y = x$ and $y = -x$. As x gets very negative the basic hyperbola gets closer and closer to $y = x$ and $y = -x$.

Asymptotes:

$y = x$	is an asymptote of the basic hyperbola as $x \to +\infty$
$y = -x$	is an asymptote of the basic hyperbola as $x \to +\infty$
$y = x$	is an asymptote of the basic hyperbola as $x \to -\infty$
$y = -x$	is an asymptote of the basic hyperbola as $x \to -\infty$.

Example: Graph $y = \frac{1}{x}$.



The graph of $y = \frac{1}{x}$

(a) As x gets large $\frac{1}{x}$ gets closer and closeer to 0.
(b) As x gets closer to 0 (from the positive side) then $\frac{1}{x}$ gets larger and larger.
(c) As x gets closer to 0 (from the negative side) then $\frac{1}{x}$ gets more and more negative.
(d) As x gets more and more negative $\frac{1}{x}$ gets closer and closer to 0.
(e) If $x = 1$ then $y = 1$.
(f) If $x = -1$ then $y = -1$.

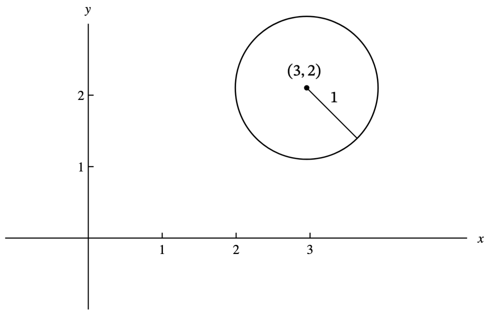
Asymptotes:

$y = 0$	(the x axis) is an asymptote to $y = \frac{1}{x}$ as $x \to +\infty$
$y = 0$	(the x axis) is an asymptote to $y = \frac{1}{x}$ as $x \to -\infty$
$x = 0$	(the y axis) is an asymptote to $y = \frac{1}{x}$ as $x \to 0^+$
$x = 0$	(the y axis) is an asymptote to $y = \frac{1}{x}$ as $x \to 0^-$.

6.8 Graphing: Shifting, scaling and flipping

6.8.1 Shifting

Example:. Graph $(x-3)^2+(y-2)^2=1$.

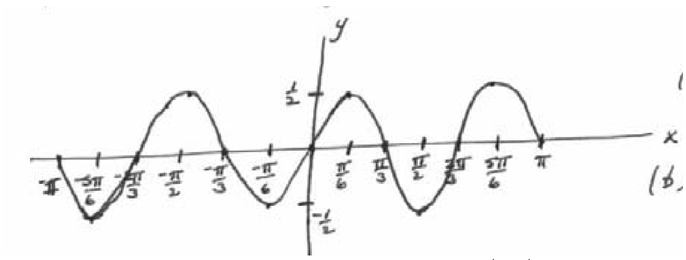


A circle of radius 1 and center $(3,2)$

To graph $(x-3)^2+(y-2)^2=1$:
(a) $x^2+y^2=1$ is a basic circle of radius 1.
(b) The center is shifted by
3 to the right in the x-direction,
2 upwards in the y-diection.

6.8.2 Scaling

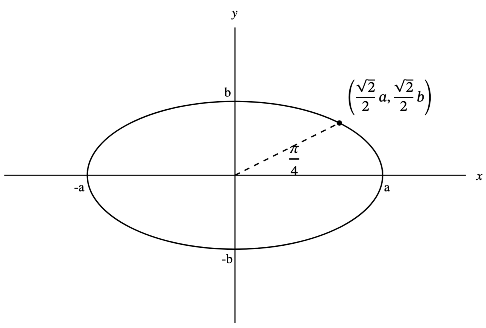
Example:. Graph $2y=\sin 3x$.



The graph of $2y=\sin(3x)$

To graph $2y=\sin(3x)$:
(a) $y=\sin x$ is the basic graph.
(b) The x-axis is scaled (squished) by 3.
(c) The y-axis is scaled by 2.

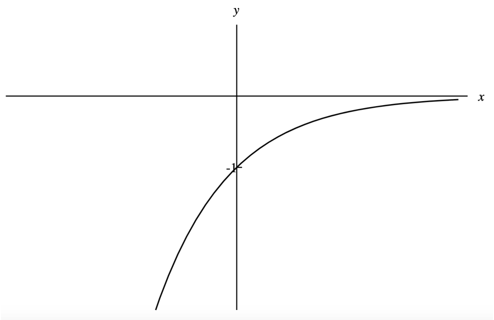
Example: Let $a,b\in\mathbb{R}_{>0}$. Graph $\frac{1}{a^2}x^2+\frac{1}{b^2}y^2=1$.



An ellipse with width $2a$ and height $2b$

6.8.3 Flipping

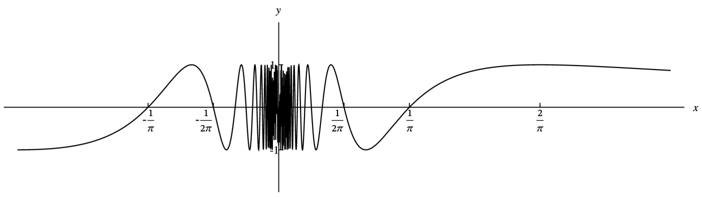
Example:. Graph $y = -e^{-x}$.



The graph of $y = -e^{-x}$

To graph $y = -e^{-x}$:
(a) $y = e^x$ is the basic graph.
(b) $y = -e^{-x}$ is the same as $-y = e^{-x}$.
(c) The x-axis is flipped (around $x = 0$).
(d) The y-axis is flipped (around $y = 0$).

Example:. Graph $y = \sin\left(\frac{1}{x}\right)$.

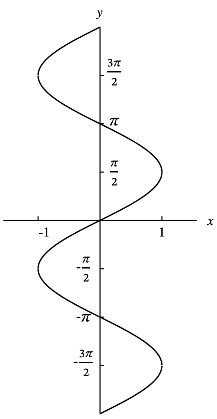


The graph of $y = \sin\left(\frac{1}{x}\right)$

To graph $y = \sin\left(\frac{1}{x}\right)$:

(a) $y = \sin x$ is the basic graph.
(b) The positive x axis is flipped (around $x = 1$).
(c) The negative x axis is flipped (around $x = -1$).
(d) As $x \to \infty$ then $\sin\left(\frac{1}{x}\right) = 0^+$.
(e) As $x \to -\infty$ then $\sin\left(\frac{1}{x}\right) = 0^-$.
(f) As $x \to 0^+$ then $\sin\left(\frac{1}{x}\right)$ oscillates between $+1$ and -1.

Example:. Graph $y = \arcsin x$.



The graph of $y = \arcsin x$

To graph $y = \arcsin x$:
(a) $y = \sin x$ is the basic graph.
(b) $y = \arcsin, x$ is the same as $\sin y = x$.
So the x and y axis are switched from $y = \sin x$.
So the graph of $y = \sin x$ is flipped across the line $x = y$.