3 Sets and functions

3.1 Sets

A *set* is a collection of objects which are called *elements*.
Write

$$s \in S \text{ if } s \text{ is an element of the set } S.$$

- The *empty set* $\emptyset$ is the set with no elements.
- A *subset* T of a set S is a set T such that if $t \in T$ then $t \in S$.

Write

$$T \subseteq S \text{ if } T \text{ is a subset of } S\text{, and}$$
$$T = S \text{ if the set } T \text{ is equal to the set } S.$$

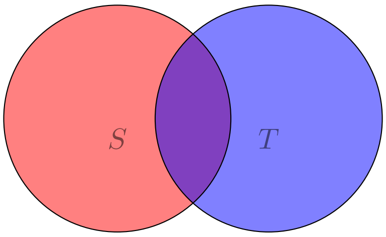
More precisely, $T = S$ if $T \subseteq S$ and $S \subseteq T$.

Let S and T be sets.

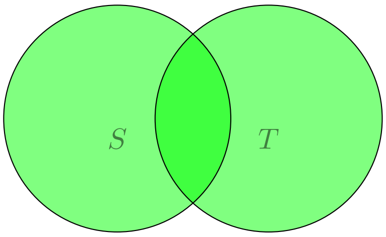
- The *union of S and T* is the set $S \cup T$ of all u such that $u \in S$ or $u \in T$,
$$S \cup T = \{u \mid u \in S \text{ or } u \in T\}.$$
- The *intersection of S and T* is the set $S \cup T$ of all u such that $u \in S$ and $u \in T$,
$$S \cap T = \{u \mid u \in S \text{ and } u \in T\}.$$
- The *product S and T* is the set $S \times T$ of all ordered pairs (s, t) where $s \in S$ and $t \in T$,
$$S \times T = \{(s, t) \mid s \in S \text{ and } t \in T\}.$$

The sets S and T are *disjoint* if $S \cap T = \emptyset$.
The set S is a *proper subset* of T if $S \subseteq T$ and $S \neq T$. Write

$$S \subsetneq T \qquad \text{if } S \text{ is a proper subset of } T.$$



The red (and purple) region is S
The blue (and purple) region is T
the purple region is $S \cap T$



the green region is $S \cup T$