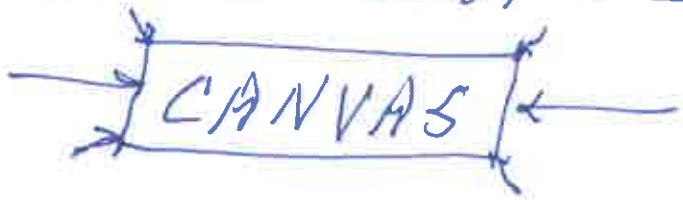
Numbers, Sets and Graphing

03.03.2025 (1)
Calculus Lect. 1
A. Ram

Assessment

Assignments - weekly 15%
Online quiz - probably in week 4, 10%
Midsemester test - probably in week 9, 15%
Final Exam 60%, in Exam period.



Example 1.1 Which numbers are further apart

π and -3 or 0 and $\sqrt{35}$?

The distance between π and -3 is

$$|-3-\pi| = |-6.1415\ldots| = 6.1415\ldots > 6.$$

the distance between 0 and $\sqrt{35}$ is

$$|\sqrt{35}-0| = |\sqrt{35}| < |\sqrt{36}| = 6$$

So π and -3 are further apart than 0 and $\sqrt{35}$.

Example 1.3 a

Math: $A = \{x \in \mathbb{R} \mid x^2 + 1 > 37\}$

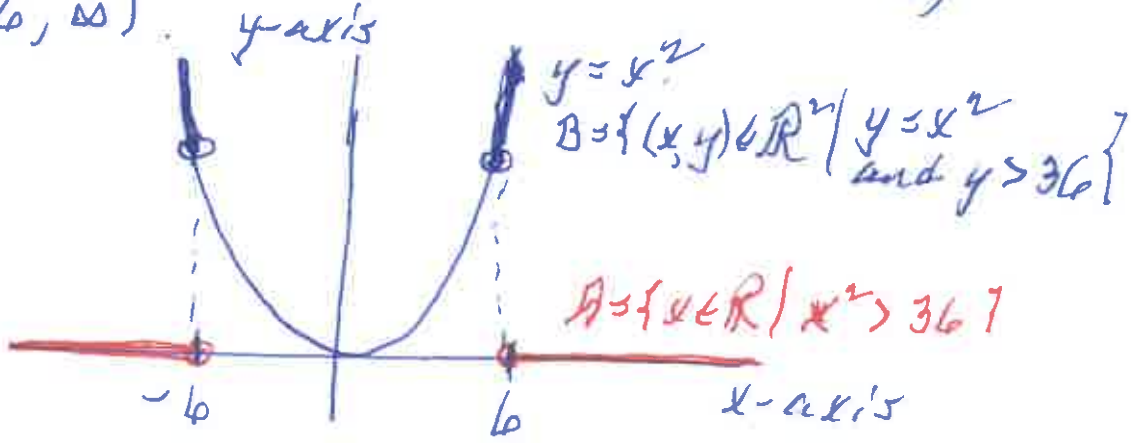
$= \{x \in \mathbb{R} \mid x^2 > 36\}$

$= \{x \in \mathbb{R} \mid x < -6\} \cup \{x \in \mathbb{R} \mid x > 6\}$

$= (-\infty, -6) \cup (6, \infty) = \mathbb{R}_{<-6} \cup \mathbb{R}_{>6}$

English: The set of x in $\mathbb{R}$ such that $x^2 + 1$ is strictly greater than 37 is the union of the intervals $(-\infty, -6)$ and $(6, \infty)$.

Cartoon:



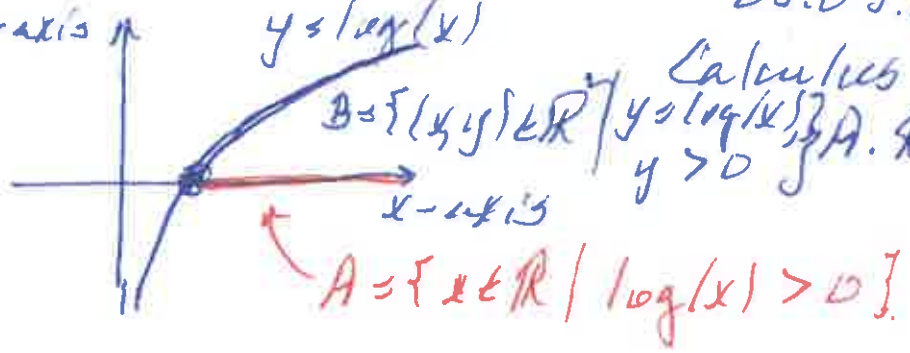
Example 1.3 b

English: The set of real numbers ~~whose~~ such that the logarithm is positive

Math: $\{x \in \mathbb{R} \mid \log(x) > 0\} = \{x \in \mathbb{R} \mid x > 1\}$

$= \mathbb{R}_{>1} = (1, \infty)$.

Cartoon:



Example 1.3c Math: $\{n \in \mathbb{N} \mid \sin(n) > 0\}$

$= \{n \in \mathbb{Z}_{>0} \mid \sin(n) > 0\}$

English: The set of strictly positive integers with strictly positive sine.

Example 1.4a The set of odd integers is

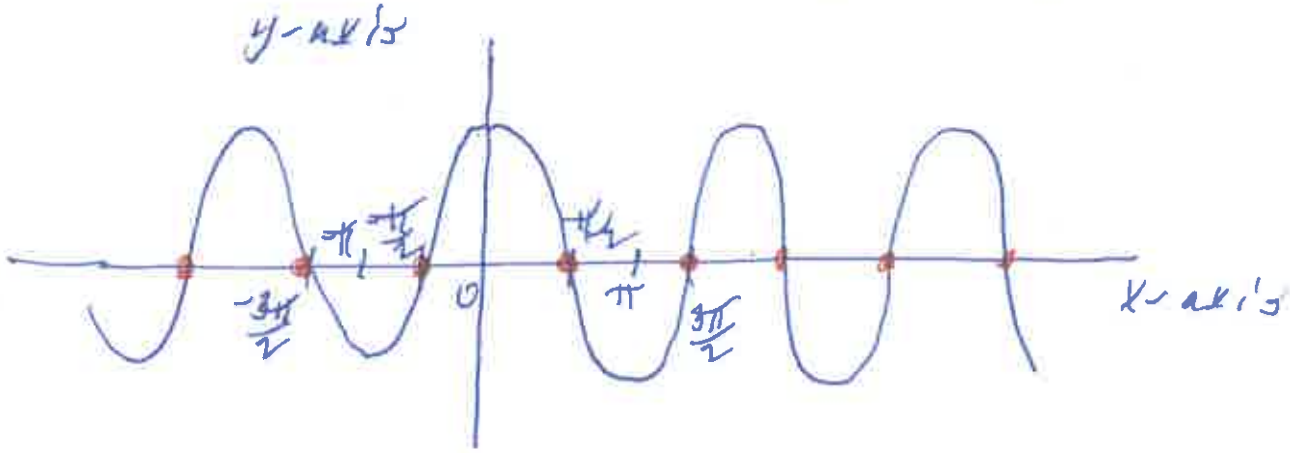
$\{x \in \mathbb{Z} \mid x \notin 2\mathbb{Z}\} = \{\ldots, -3, -1, 1, 3, \ldots\}$

$= \{2k+1 \mid k \in \mathbb{Z}\}$.

Example 1.4b

$\{x \in \mathbb{R} \mid \cos(x) = 0\} = \{\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\}$

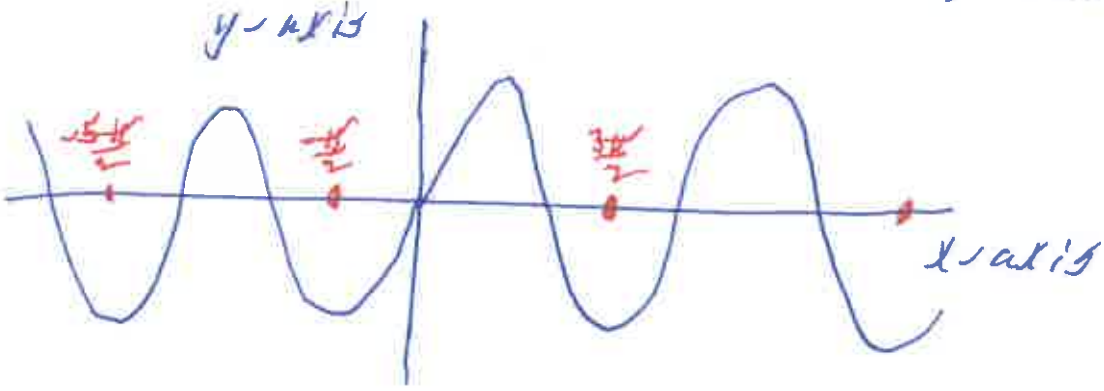
$= \{(2k+1)\frac{\pi}{2} \mid k \in \mathbb{Z}\}$



Example 1.4/c

$\{x \in \mathbb{R} / \sin(x) = -1\} = \{\ldots, -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots\}$

$= \{(2k+1)\pi / k \in \mathbb{Z}\} = \{-\frac{\pi}{2} + 2k\pi / k \in \mathbb{Z}\}$.



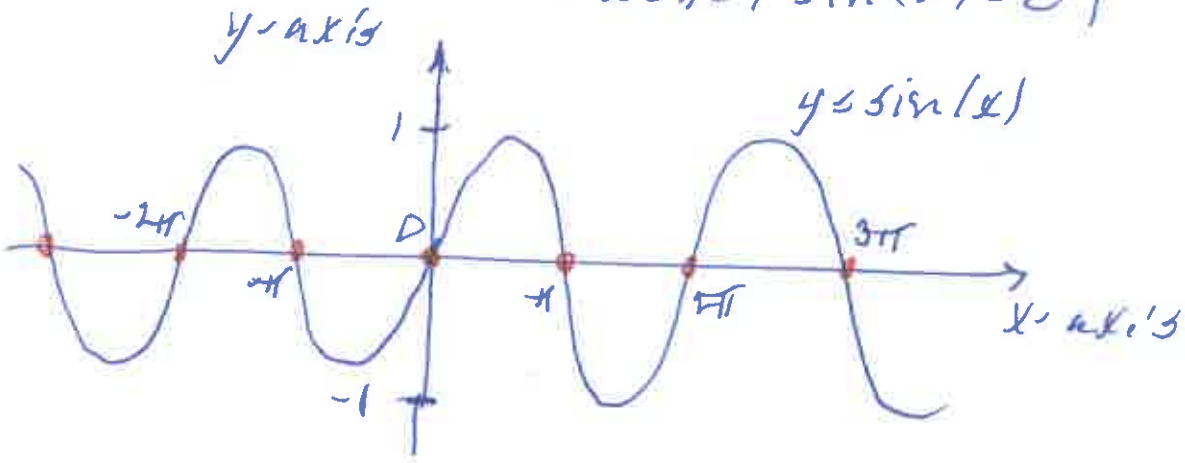
Example 1.6 The set of prime numbers less than 20 is

$\{x \in \mathbb{Z}_{>0} / x \text{ is prime and } x \in 20\}$

$= \{2, 3, 5, 7, 11, 13, 17, 19\}$

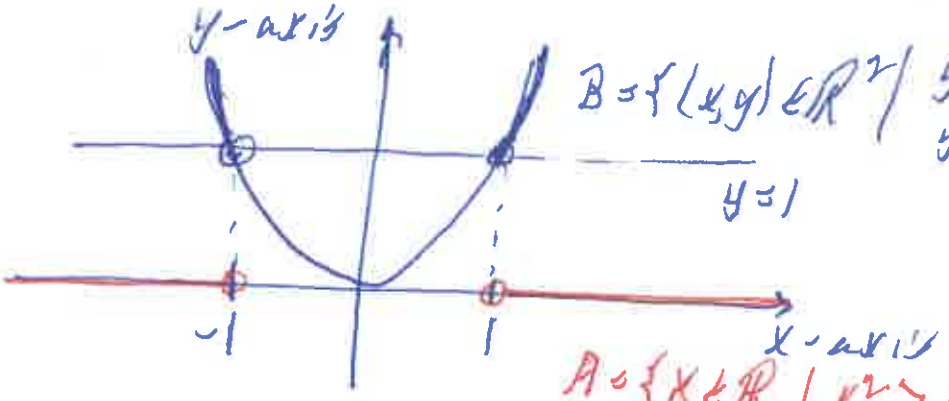
Example 1.7 $\{k\pi / k \in \mathbb{Z}\} = \{\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$

$= \{x \in \mathbb{R} / \sin(x) = 0\}$



Example 1.12a As a union of intervals, A. Rau

$$\{x \in \mathbb{R} \mid x^2 > 1\} = (-\infty, -1) \cup (1, \infty)$$

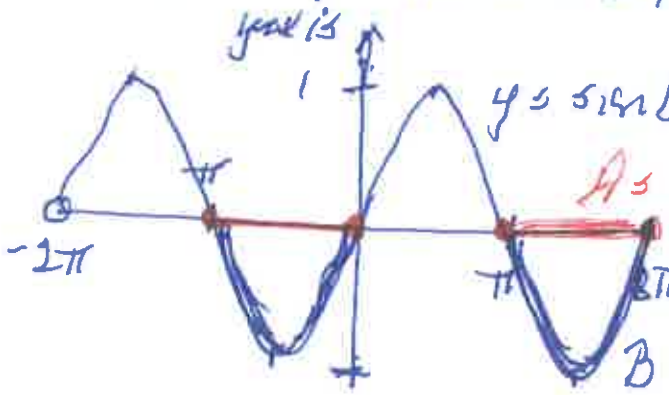


$$B = \{(x,y) \in \mathbb{R}^2 \mid y = x^2 \text{ and } y > 1\}$$

$$A = \{x \in \mathbb{R} \mid x^2 > 1\}$$

Example 1.12b As a union of intervals

$$\{x \in (-2\pi, 2\pi] \mid \sin(x) \leq 0\} = [-\pi, 0] \cup [\pi, 2\pi]$$



$$A = \{x \in (-2\pi, 2\pi] \mid \sin(x) \leq 0\}$$

$$B = \{(x,y) \in \mathbb{R}^2 \mid y = \sin(x) \text{ and } y \leq 0 \text{ and } x \in (-2\pi, 2\pi]\}$$

Example 1.12c As a union of intervals,

$$\{x \in [-2, 2] \mid x \notin \mathbb{Z}\} = (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (1, 2)$$