05.03.2025

Calculus Lect. 2 ①

A. Ram.

To do list:

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

A union B is
A and B together

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

A intersect B is
where A and B overlap

$A \setminus B = \{x \in A \mid x \notin B\}$

A minus B is stuff
in A that is not
in B

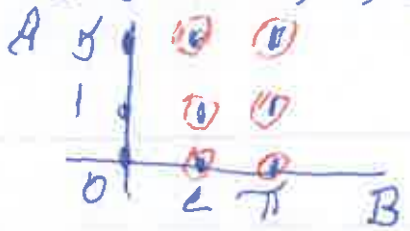
$A \times B = \{(x, y) \mid x \in A, y \in B\}$

A product B is
pairs with the first
from A and the second
from B

Example 1.2.1

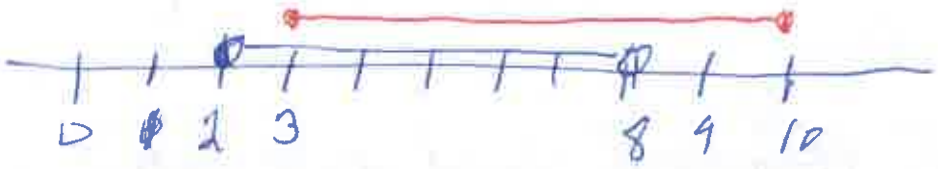
If $A = \{0, 1, 5\}$ and
$B = \{e, \pi\}$ then

$A \times B = \{(0, e)\ (0, \pi)$
$(1, e)\ (1, \pi)$
$(5, e)\ (5, \pi)\}$



Example 1.13a

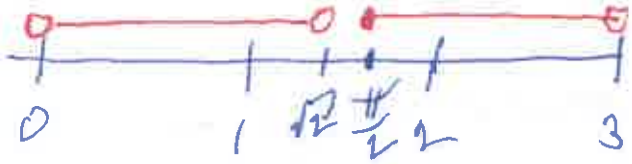
$$(2,8) \cup [3,10] = \{x \in \mathbb{R} \mid 2 < x < 8\} \cup \{x \in \mathbb{R} \mid 3 \le x \le 10\}$$
$$= \{x \in \mathbb{R} \mid 2 < x \le 10\} = (2,10]$$



$(2,8) \cap [3,10] = [3,8)$ (Example 1.15a)

Example 1.13b

$[0, \sqrt{2}) \cup [\frac{\pi}{2}, 3)$ is not connected and not an interval



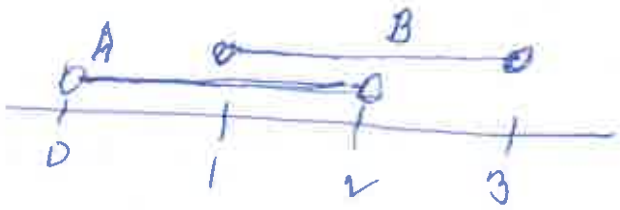
$\frac{\pi}{2} > 1.5$ and $\sqrt{2} = 1.414\ldots$

$(0, \sqrt{2}) \cap [\frac{\pi}{2}, 3) = \phi$ (Example 1.15b)

Example 1.18 $A=(0,2]$ and $B=(1,3)$

$A \setminus B = (0,2] \setminus (1,3) = (0,1]$

$B \setminus A = (1,3) \setminus (0,2] = [2,3)$



This is an example of when $A \setminus B \neq B \setminus A$.

Intersections (Examples 1.15 c and d)

$\mathbb{Z} \cap [-\pi, \pi] = \{-3,-2,-1,0,1,2,3\}$

$= \{x \in \mathbb{R} \mid x \in \mathbb{Z} \text{ and } -\pi \leq x \leq \pi\}$

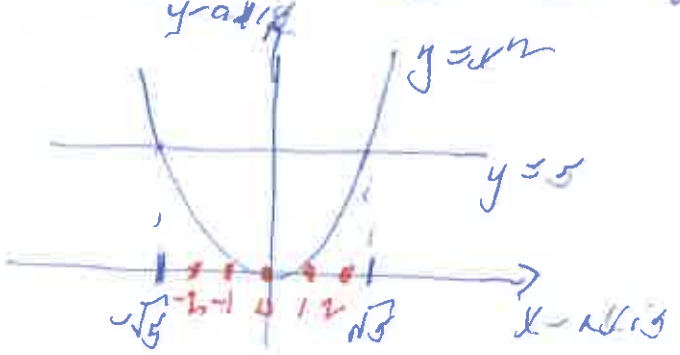
$= \{x \in \mathbb{Z} \mid -3.14\ldots \leq x \leq 3.14\ldots\}$

$\mathbb{Z} \cap \{x \in \mathbb{R} \mid x^2 - 5 < 0\} = \mathbb{Z} \cap \{x \in \mathbb{R} \mid x^2 < 5\}$

$= \{x \in \mathbb{Z} \mid -\sqrt{5} < x < \sqrt{5}\}$

$= \{x \in \mathbb{Z} \mid -3 < x < 3\} = \{-2,-1,0,1,2\}$

since $2 < \sqrt{5} < 3$ and $-3 < -\sqrt{5} < -2$



Intersections (Example 1.16)

$A = \{x \in \mathbb{R} \mid \sin|x| > 0 \text{ and } \cos|x| < 0\}$

$= \{x \in \mathbb{R} \mid \sin(x) > 0\} \cap \{x \in \mathbb{R} \mid \cos(x) < 0\}$

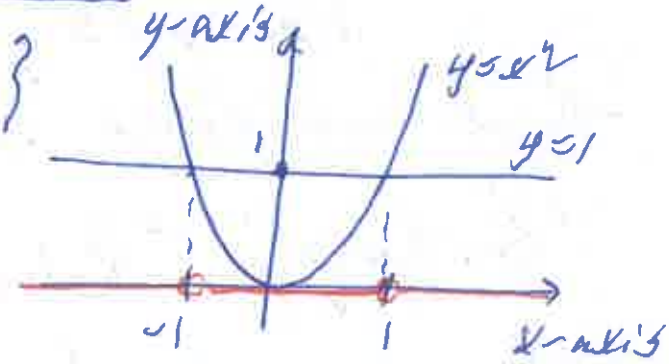
is the set of real numbers with positive sine and negative cosine.

$B = \{n \in \mathbb{Z} \mid \sin|n| > 0\} = \mathbb{Z} \cap \{x \in \mathbb{R} \mid \sin(x) > 0\}$

is the set of integers with positive sine.

Unions of intervals (Example 1.19) and differences/complements

$\left\{x \in \mathbb{R} \mid \frac{x^5 - 7x + 4}{x^2 - 1} \text{ is defined}\right\}$

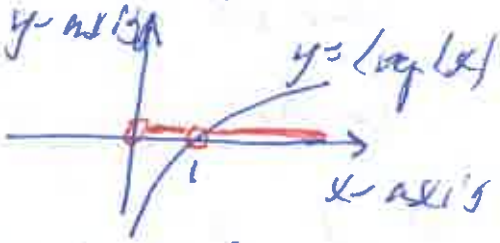


$= \{x \in \mathbb{R} \mid x^2 - 1 \neq 0\}$

$= \{x \in \mathbb{R} \mid x^2 \neq 1\}$

$= \mathbb{R} \setminus \{-1, 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

$\left\{x \in \mathbb{R} \mid \frac{\log(x)}{x^2 - 1} \text{ is defined}\right\}$



$= \{x \in \mathbb{R} \mid \log(x) \text{ is defined and } x^2 - 1 \neq 0\}$

$= \{x \in \mathbb{R}_{>0} \mid x^2 \neq 1\}$

$= \mathbb{R}_{>0} \setminus \{-1, 1\} = (0, 1) \cup (1, \infty) = \mathbb{R}_{>0} \setminus \{1\}$.