06.03.2012 (1)
Calculus Lect. 3
A. Ram

## List of elements

**Example 1.6** The set of prime numbers less than 20 is

$$\{x \in \mathbb{Z}_{>0} \mid x \text{ is prime and } x < 20\}$$
$$= \{2, 3, 5, 7, 11, 13, 17, 19\}$$

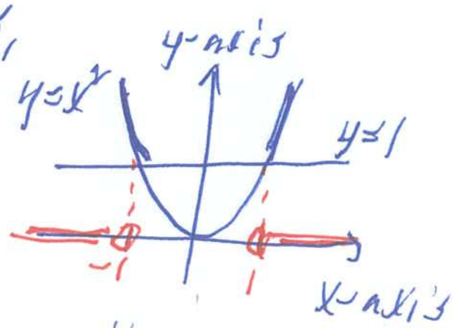
**Example 1.7**

$$\{k\pi \mid k \in \mathbb{Z}\} = \{\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$$

## Unions of intervals

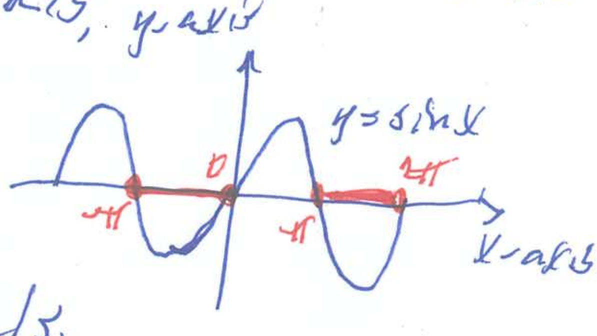
**1.12a** As a union of intervals,

$$\{x \in \mathbb{R} \mid x^2 > 1\} = (-\infty, -1) \cup (1, \infty)$$



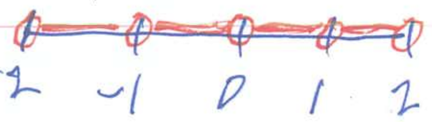

**1.12b** As a union of intervals,

$$\{x \in [-2\pi, 2\pi] \mid \sin(x) \leq 0\}$$
$$= [-\pi, 0] \cup [\pi, 2\pi]$$




**1.12c** As a union of intervals,

$$\{x \in [-2, 2] \mid x \notin \mathbb{Z}\} = (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (1, 2)$$



06.03.2015 (2)
Calculus Lect 3
A. Ram

<u>Intersections</u> (Example 1.16)

$A = \{x \in \mathbb{R} \mid \sin(x) > 0 \text{ and } \cos(x) < 0\}$

$\quad = \{x \in \mathbb{R} \mid \sin(x) > 0\} \cap \{x \in \mathbb{R} \mid \cos(x) < 0\}$

is the set of real numbers with positive sine and negative cosine.

$B = \{n \in \mathbb{Z} \mid \sin(n) > 0\} = \mathbb{Z} \cap \{x \in \mathbb{R} \mid \sin(x) > 0\}$

is the set of integers with positive sine.

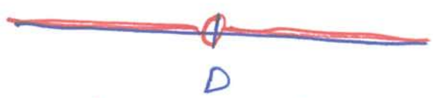
<u>Complements</u> (Example 1.20) (see Example 1.12)

(a) $\{x \in \mathbb{R} \mid x^2 > 1\} = (-\infty, -1) \cup (1, \infty)$

$\quad = \mathbb{R} - [-1, 1] = \mathbb{R} \setminus [-1, 1]$.

(b) $\{x \in [-2, 2] \mid x \notin \mathbb{Z}\} = (-2, 2) \setminus \{-1, 0, 1\}$

(c) $(-\infty, 0) \cup (0, \infty) = \mathbb{R} \setminus \{0\}$

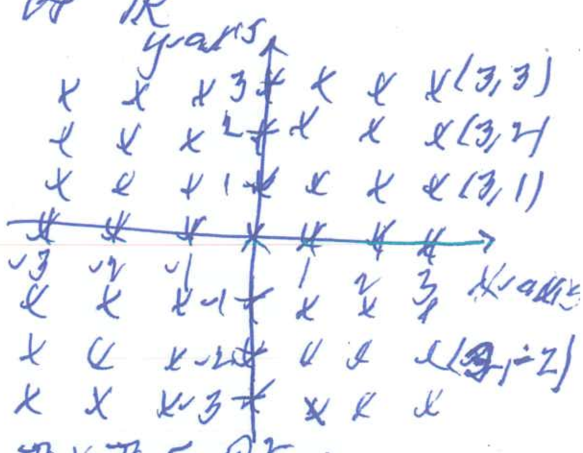



<u>Products</u> (Example 1.22)

Sketch $\mathbb{Z} \times \mathbb{Z}$ as a subset of $\mathbb{R}^2$

$\mathbb{Z} \times \mathbb{Z} = \{(x, y) \mid x \in \mathbb{Z} \text{ and } y \in \mathbb{Z}\}$

$\mathbb{R}^2 = \{(x, y) \mid x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$




$\mathbb{Z} \times \mathbb{Z} \subseteq \mathbb{R}^2$.

06.03.2015 (3)
Calculus Lect 3
A. Ram

## Subsets and equality of sets

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}.$$

Every positive integer is an integer.
Every integer is a rational number.
Every rational number is a real number.
Every real number is a complex number.

Since
$i \in \mathbb{C}$ and $i \notin \mathbb{R}$ then $\mathbb{R} \neq \mathbb{C}$,
Since $\sqrt{2}$ and $\pi$ are in $\mathbb{R}$ and not in $\mathbb{Q}$ then $\mathbb{Q} \neq \mathbb{R}$
Since $\frac{1}{2} \in \mathbb{Q}$ and $\frac{1}{2} \notin \mathbb{Z}$ then $\mathbb{Q} \neq \mathbb{Z}$
Since $-1 \in \mathbb{Z}$ and $-1 \notin \mathbb{N}$ then $\mathbb{Z} \neq \mathbb{N}$.

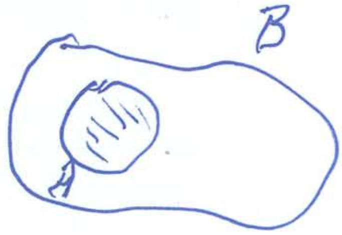
So
$$\mathbb{N} \subsetneqq \mathbb{Z} \subsetneqq \mathbb{Q} \subsetneqq \mathbb{R} \subsetneqq \mathbb{C}.$$

<u>Example 1.10</u> $A = \{2k \mid k \in \mathbb{Z}\} = 2\mathbb{Z}$.
Then
$$2\mathbb{Z} \subseteq \mathbb{Z} \quad \text{(even integers are integers)}.$$
Since $3 \in \mathbb{Z}$ and $3 \notin 2\mathbb{Z}$ then $\mathbb{Z} \neq 2\mathbb{Z}$.
So $2\mathbb{Z} \subsetneqq \mathbb{Z}$.

Let $A$ and $B$ be sets.

$A$ is a <u>subset</u> of $B$ if $A$ and $B$ satisfy

if $a \in A$ then $a \in B$




$A$ is <u>equal</u> <u>to $B$</u> if

$A \subseteq B$ and $B \subseteq A$

<u>Example 1.24</u> Let

$A = \{4n \mid n \in \mathbb{Z}\}$ and $B = \{2m+2 \mid m \in \mathbb{Z}\}$.

Prove that $A \subseteq B$.

<u>Solution</u>: To show: $A \subseteq B$

To show: If $a \in A$ then $a \in B$.

Assume $a \in A$.

To show: $a \in A$.

Since $a \in A$ then there exists $n \in \mathbb{Z}$ such that $a = 4n$.

So $a = 4n = 2 \cdot 2n = 2(2n-1)+2$.

Since multiplying integers gives an integer and subtracting 1 from an integer gives an integer then $2n-1 \in \mathbb{Z}$. So $a = 2 \cdot (2n-1) + 2 \in B$.

So $a \in B$.

So $A \subseteq B$ //.