17.03.2025
Calculus Lect 6, (1)
A. Ram

The exponential

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \dots$$

$$= 1 + x + \frac{1}{2\cdot 1}x^2 + \frac{1}{3\cdot 2\cdot 1}x^3 + \frac{1}{4\cdot 3\cdot 2\cdot 1}x^4 + \frac{1}{5\cdot 4\cdot 3\cdot 2\cdot 1}x^5 + \dots$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \dots$$

Theorem If $xy = yx$ then

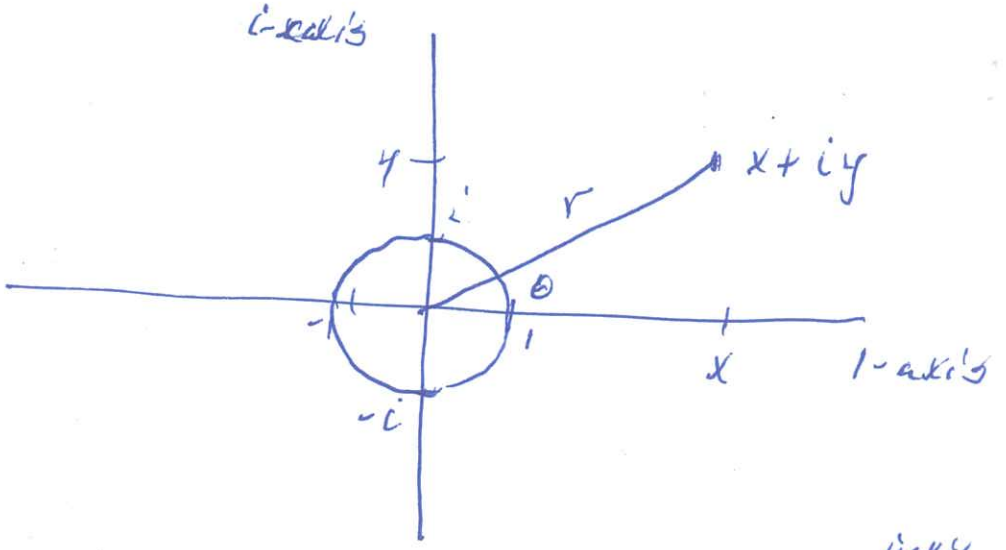
$$e^{x+y} = e^x e^y$$

Complex numbers

$$\mathbb{C} = \{x + iy \mid x, y \in \mathbb{R}\}$$

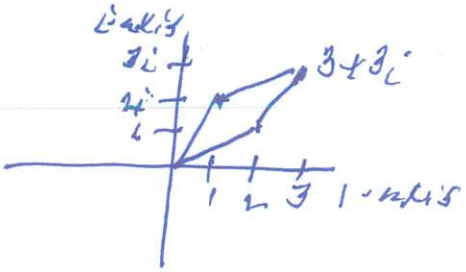
$$= \{r e^{i\theta} \mid r \in \mathbb{R}_{>0}, \theta \in \mathbb{R}\} \cup \{0\}$$

with $e^{i(\theta + 2\pi)} = e^{i\theta}$ (or $e^{i2\pi} = e^0$)



2.8 Addition

$$(1 + 2i) + (2 + i) = 3 + 3i$$



2.9 Multiplication

$(4+2i)(5+3i) = 20+12i+10i+6i^2$

$= 20+22i-6 = 14+22i.$

2.15 Division

$(a+b)(a-b) = a^2-b^2$

2.16

$(x+iy)(x-iy) = x^2+y^2 = r^2$

$$\frac{1+2i}{-1+3i} = \frac{(1+2i)(-1-3i)}{(-1+3i)(-1-3i)} = \frac{-1-3i-2i-6i^2}{1+9} = \frac{-1-5i+6}{10}$$

$$= \frac{+5-5i}{10} = \frac{1}{2} - \frac{1}{2}i.$$

2.10 Conjugate

$z = x+iy$ has $\bar{z} = x-iy$

$\overline{-3+7i} = -3-7i$; $\overline{2-5i} = 2+5i$, $\overline{3i} = -3i$; $\overline{4} = 4$

2.11 Length

$|z| = \sqrt{x^2+y^2} = r$, so $|re^{i\theta}| = r$

$|-2| = 2$, $|3-i| = \sqrt{9+1} = \sqrt{10}$, $|5+2i| = \sqrt{25+4} = \sqrt{29}$

$|1+3i| = \sqrt{1+9} = \sqrt{10}$ $|7-i| = \sqrt{49+1} = \sqrt{50}$

and

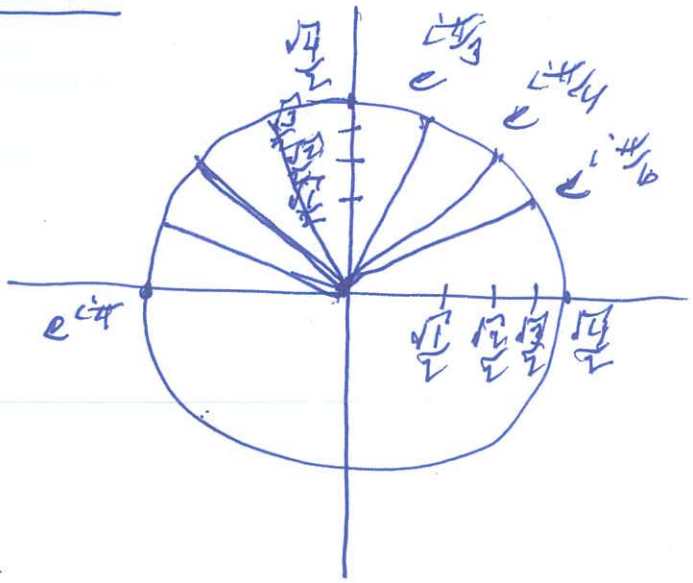
$$\left|\frac{-2(3-i)(5+2i)}{(1+3i)(7-i)}\right| = \frac{2\sqrt{10}\cdot\sqrt{29}}{\sqrt{10}\cdot\sqrt{50}} = \frac{2\sqrt{29}}{5\sqrt{2}}$$

since, if $z = re^{i\theta}$ and $w = se^{i\beta}$ then

$|zw| = |re^{i\theta}se^{i\beta}| = |rse^{i(\theta+\beta)}| = rs = |z||w|$

17.03.2025
Calculus Lect. 6 (3)
A. Ran

Polar Form



The TATTOO

2.19 $1+\sqrt{3}i = 2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 2\cdot e^{i\pi/3}$

$-3-3i = \frac{-3}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\cdot 2 = -3\sqrt{2}\,e^{i\pi/4}$

$3+4i = 5e^{i\arctan(4/3)}$

$-1 = e^{i\pi}$.

2.24 $\sqrt{3}+i = 2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = 2e^{i\pi/6}$

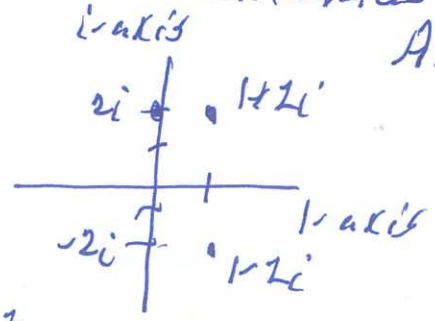
$-1-i = \frac{-2}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = \frac{-2}{\sqrt{2}}e^{i\pi/4} = -\sqrt{2}e^{i\pi/4} = \sqrt{2}e^{i\frac{5\pi}{4}}$

2.27 $5e^{i\frac{3\pi}{4}} = 5\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = -\frac{5\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}i$.

2.46 $i^7 = i\cdot i\cdot i\cdot i\cdot i\cdot i\cdot i = (-1)\cdot(-1)\cdot(-1)\cdot i = -i$

2.6 $\mathrm{Re}(2-3i) = 2$ and $\mathrm{Im}(2-3i) = -3$.

Graphing



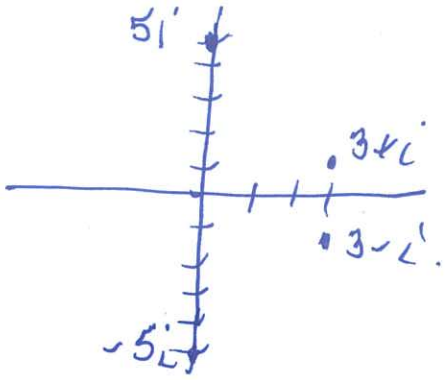
2.1 $\{x \in \mathbb{C} \mid x^2+4=0\}$

$= \{2i, -2i\}$

since $(2i)^2 = 4i^2 = -4$ and $(-2i)^2 + 4 = 4i^2 + 4 = -4+4 = 0$

2.1 $\{x \in \mathbb{C} \mid x^2-2x+5=0\}$

$= \{1+2i, 1-2i\}$



2.4 $\{x \in \mathbb{C} \mid x^2 = -25\}$

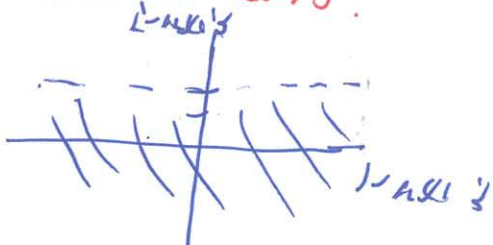
$= \{5i, -5i\}$

2.13 $\{z \in \mathbb{C} \mid z^2-6z+10=0\}$

$= \{3+i, 3-i\}$

2.11 $\bar{z}$ is reflection of z in the 1-axis.

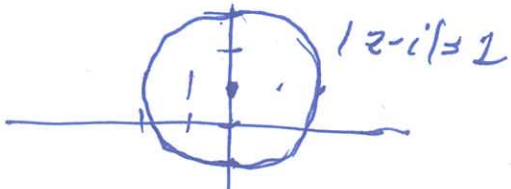
2.36 $\{z \in \mathbb{C} \mid \mathrm{Im}(z) < 2\}$



2.37 $\{z \in \mathbb{C} \mid |z-i| = 2\}$

$= \{z \in \mathbb{C} \mid$ length of $z-i$ is $2\}$

$= \{z \in \mathbb{C} \mid$ distance from z to i is $2\}$



2.38 $\{z \in \mathbb{C} \mid |z-3-2i| < 3\}$

$= \{z \in \mathbb{C} \mid$ length of $z-(3+2i)$ is $< 3\}$

$= \{z \in \mathbb{C} \mid$ distance from z to $-3+2i < 3\}$