09.04.2025
Calculus Lect 16 (1)
A. Ram

We had

$$\frac{d\sin(y)}{dx} = \cos(y)\frac{dy}{dx}$$

or $$\frac{d\sin(JUNK)}{dx} = \cos(JUNK)\frac{d(JUNK)}{dx}.$$

4.10 $$\frac{d}{dx}(\sin(3x^2+8)) = \cos(3x^2+8)\frac{d(3x^2+8)}{dx}$$

$$= \cos(3x^2+8)(3\cdot 2x+0) = 6x\cos(3x^2+8)$$

4.8 $$\frac{d\tan(y)}{dx} = \frac{d}{dx}\left(\frac{\sin(y)}{\cos(y)}\right) = \frac{d}{dx}(\sin(y)\cdot\cos(y)^{-1})$$

$$= \sin(y)\frac{d\cos(y)^{-1}}{dx} + \frac{d\sin(y)}{dx}\cos(y)^{-1}$$

$$= \sin(y)(-1)\cos(y)^{-2}\frac{d\cos(y)}{dx} + \cos(y)\frac{1}{\cos(y)}\frac{dy}{dx}$$

$$= \frac{-\sin(y)}{\cos(y)^2}(-\sin(y))\frac{dy}{dx} + \frac{dy}{dx}$$

$$= \left(\frac{\sin(y)^2}{\cos(y)^2} + 1\right)\frac{dy}{dx} = \frac{\sin(y)^2+\cos(y)^2}{\cos(y)^2}\cdot\frac{dy}{dx}$$

$$= \frac{1}{\cos(y)^2}\frac{dy}{dx} = \sec(y)^2\frac{dy}{dx}.$$

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4.18 Let
$$\cosh(y) = \frac{1}{2}(e^y + e^{-y})$$
$$\sinh(y) = \frac{1}{2}(e^y - e^{-y})$$
Then
$$\frac{d\cosh(y)}{dx} = \frac{d}{dx}\left(\frac{1}{2}e^y + \frac{1}{2}e^{-y}\right) = \frac{1}{2}\frac{de^y}{dx} + \frac{1}{2}\frac{de^{-y}}{dx}$$
$$= \frac{1}{2}e^y\frac{dy}{dx} + \frac{1}{2}e^{-y}\frac{d(-y)}{dx}$$
$$= \frac{1}{2}e^y\frac{dy}{dx} + \frac{1}{2}e^{-y}(-1)\frac{dy}{dx} = \frac{1}{2}(e^y - e^{-y})\frac{dy}{dx}$$
$$= \sinh(y)\frac{dy}{dx}$$
and
$$\frac{d\sinh(y)}{dx} = \frac{d}{dx}\left(\frac{1}{2}(e^y - e^{-y})\right) = \frac{1}{2}\left(e^y\frac{dy}{dx} - e^{-y}\frac{d(-y)}{dx}\right)$$
$$= \frac{1}{2}(e^y + e^{-y})\frac{dy}{dx} = \cosh(y)\frac{dy}{dx}.$$

4.34a
$$\frac{d}{dx}(xe^y) = x\frac{de^y}{dx} + \frac{dx}{dx}e^y$$
$$= xe^y\frac{dy}{dx} + 1\cdot e^y = xe^y\frac{dy}{dx} + e^y.$$

4.34d

$$\frac{d}{dx}(x^2\log(y)) = x^2\frac{d}{dx}(\log(y)) + \frac{dx^2}{dx}\cdot\log(y)$$

$$= x^2\frac{1}{y}\frac{dy}{dx} + 2x\log(y) = \frac{x^2}{y}\frac{dy}{dx} + 2x\log(y).$$

4.35a Let $x^4+y^2=1+x^2y$. Find $\frac{dy}{dx}$.

Take $\frac{d}{dx}$ of both sides to get

$$4x^3+2y\frac{dy}{dx} = 0 + \frac{d(x^2y)}{dx}$$

So $$4x^3+2y\frac{dy}{dx} = x^2\frac{dy}{dx} + 2x\cdot y$$

So $$(2y-x^2)\frac{dy}{dx} = 2xy-4x^3.$$

So $$\frac{dy}{dx} = \frac{2xy-4x^3}{2y-x^2}.$$

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4.37 Let $y = \arcsin(x)$. Find $\frac{dy}{dx}$

Since $\sin(y) = \sin(\arcsin(x)) = x$

then $\frac{d\sin(y)}{dx} = \frac{dx}{dx}$. So $\cos(y)\frac{dy}{dx} = 1$.

So $\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{\cos(y)^2}} = \frac{1}{\sqrt{1-\sin(y)^2}}$

$= \frac{1}{\sqrt{1-x^2}}$. So $\frac{d\arcsin(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

4.38 Let $y = \arctan(x)$. Find $\frac{dy}{dx}$

Since $\tan(y) = \tan(\arctan(x)) = x$

then $\frac{d\tan(y)}{dx} = \frac{dx}{dx}$. So $\sec(y)^2\frac{dy}{dx} = 1$.

So $\frac{1}{\cos(y)^2}\frac{dy}{dx} = 1$. So $\frac{dy}{dx} = \cos(y)^2$.

So $\frac{dy}{dx} = \frac{\cos(y)^2}{\sin(y)^2+\cos(y)^2} = \frac{1}{\frac{\sin(y)^2}{\cos(y)^2} + \frac{\cos(y)^2}{\cos(y)^2}}$

$= \frac{1}{\tan(y)^2+1} = \frac{1}{x^2+1}$.

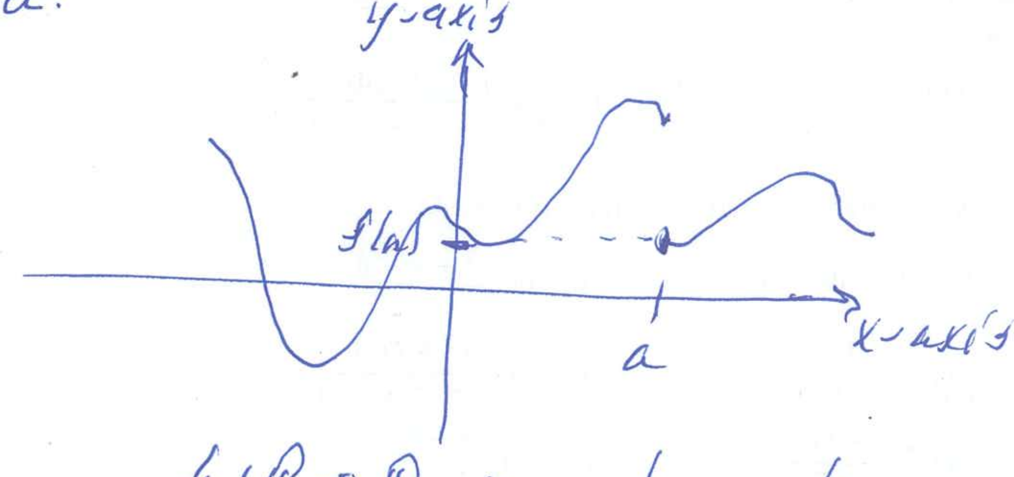
Continuity and differentiability

Let $a \in \mathbb{R}$. A function $f: \mathbb{R} \to \mathbb{R}$ is **continuous at a** if

$$\lim_{x \to a} f(x) = f(a)$$

($f(x)$ gets closer and closer to $f(a)$ as x gets closer and closer to a)

In other words, $f: \mathbb{R} \to \mathbb{R}$ is continuous at a if the graph of f doesn't jump at $x = a$.



$f: \mathbb{R} \to \mathbb{R}$ is not continuous at a

The fundamental theorem of change

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f'(a)$ means, evaluate $\frac{df}{dx}$ at $x = a$.

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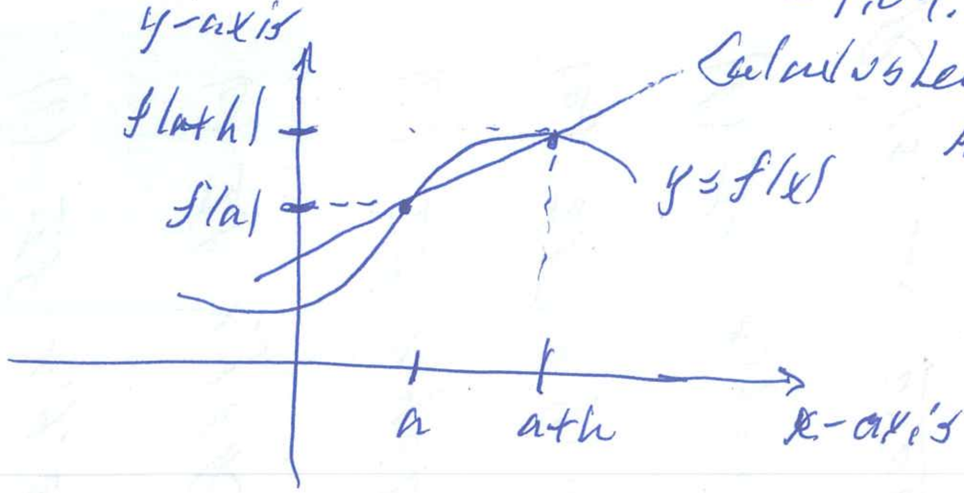

$$\frac{f(a+h)-f(a)}{h} = \frac{\text{change in } f}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

= slope of line connecting $(a, f(a))$ and $(a+h, f(a+h))$

So

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \left(\begin{array}{c}\text{slope of } f \text{ at the} \\ \text{point } x=a\end{array}\right)$$

A function is <u>differentiable</u> <u>at $x=a$</u> if the slope of the graph of $f(x)$ at $x=a$ exists.