Graphing

4.16 Let $f: [-1, \sqrt{3}] \to \mathbb{R}$ given by $f(x) = \sqrt{3}(x^3 - x)$.

Then $f(x) = \sqrt{3}\, x(x^2 - 1) = \sqrt{3}\, x(x-1)(x+1)$.

(a) $f(0) = 0$ and $f(1) = 0$ and $f(-1) = 0$

and $f(\sqrt{3}) = \sqrt{3}\left((\sqrt{3})^3 - \sqrt{3}\right) = (\sqrt{3})^4 - 3 = 9 - 3 = 6$

and $f(x) \to \infty$ as $x \to \infty$

and $f(x) \to -\infty$ as $x \to -\infty$.

(b) $f'(x) = \sqrt{3}(3x^2 - 1) = 3\sqrt{3}\left(x^2 - \frac{1}{3}\right)$

$= 3\sqrt{3}\left(x - \frac{1}{\sqrt{3}}\right)\left(x + \frac{1}{\sqrt{3}}\right)$

So $f'\left(\frac{1}{\sqrt{3}}\right) = 0$ and $f'\left(\frac{-1}{\sqrt{3}}\right) = 0$ and

f has stationary points at $x = \frac{1}{\sqrt{3}}$

and $x = \frac{-1}{\sqrt{3}}$

f is increasing on $[-1, \frac{-1}{\sqrt{3}})$ and $[\frac{+1}{\sqrt{3}}, \sqrt{3}]$

f is decreasing on $[\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

f has local minimums at $x = \frac{1}{\sqrt{3}}$ and $x = -1$

f has local maximums at $x = \frac{-1}{\sqrt{3}}$ and $x = \sqrt{3}$.

14.04.2025 (2)

Calculus test 1/8

A. Ram

f has a global maximum at $x = \sqrt{3}$.

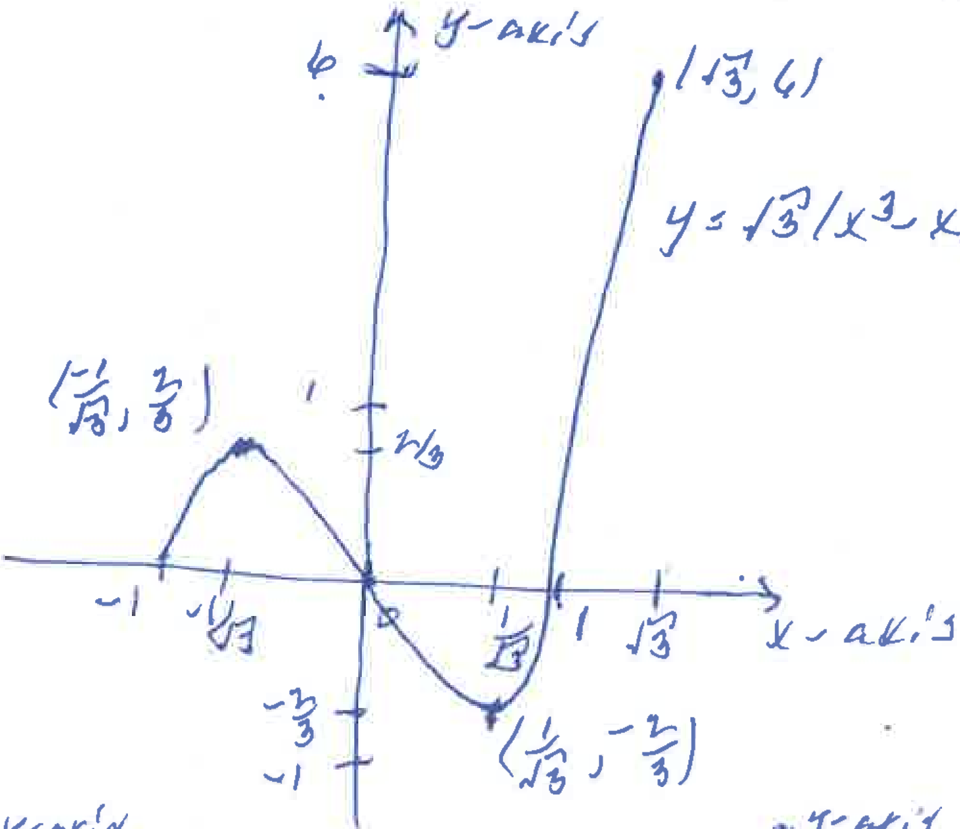
f has a global minimum at $x = \frac{1}{\sqrt{3}}$.

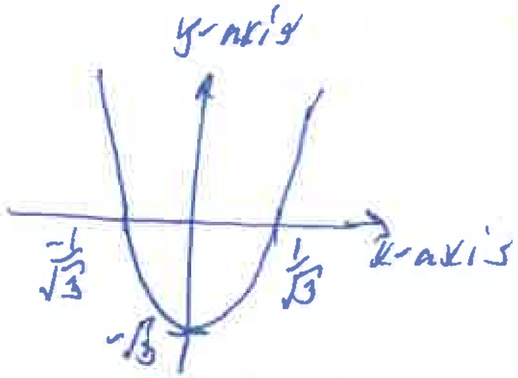
(c) $f''(x) = \sqrt{3} \cdot 6x = 6\sqrt{3}\, x$.

f is concave down on $\mathbb{R}_{<0} = (-\infty, 0)$

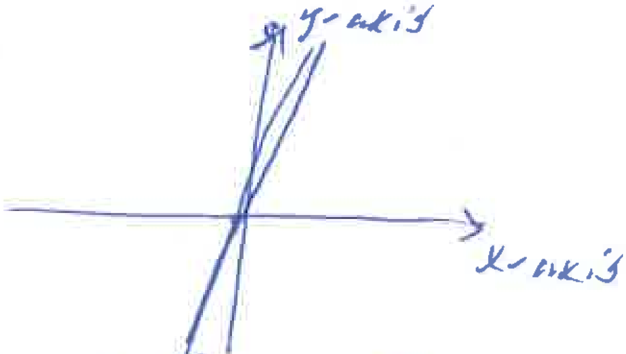
f is concave up on $\mathbb{R}_{>0} = (0, \infty)$

f has a point of inflection at $x = 0$.



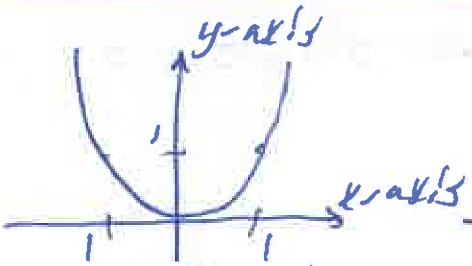


Graph of $y = \sqrt{3}(3x^2 - 1)$

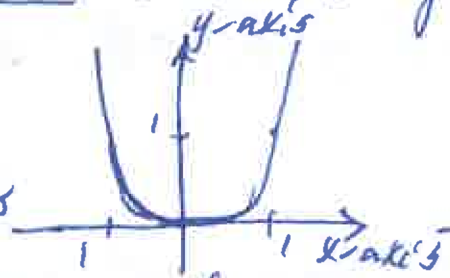


Graph of $y = 6\sqrt{3}\, x$

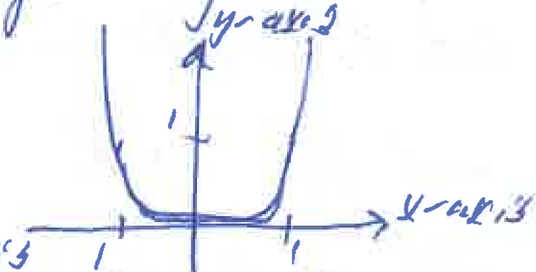
<u>4.23a and 4.25a</u> $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^4$



Graph of $y = x^2$

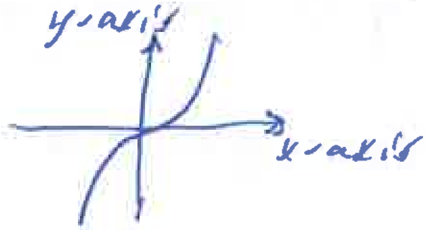


Graph of $y = x^4$



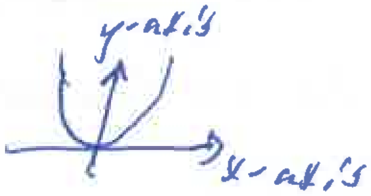
Graph of $y = x^8$

$f'(x) = 4x^3$ has graph



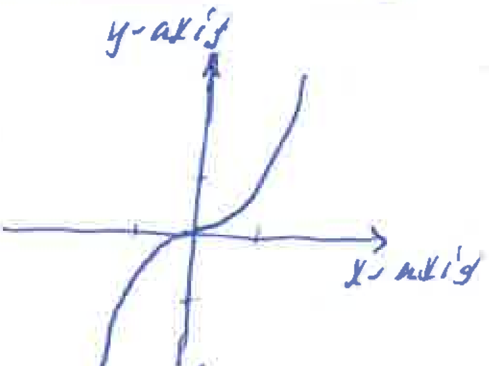
So
f is increasing on $\mathbb{R}_{>0}$
f is decreasing on $\mathbb{R}_{<0}$
f has a stationary point at $x = 0$
f has a minimum at $x = 0$.
f has no maximum.

$f''(x) = 12x^2$ has graph

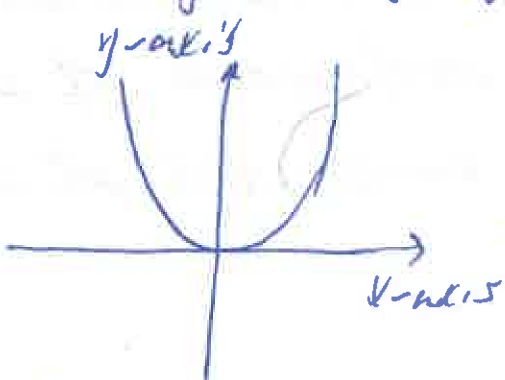


f is concave up on $\mathbb{R}$
f does <u>not</u> have a point of inflection at $x = 0$.

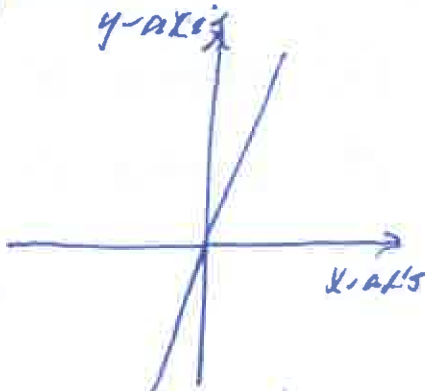
<u>4.23b and 4.25b</u> $g: \mathbb{R} \to \mathbb{R}$ given by $g(x) = x^3$.



Graph of $y = x^3$



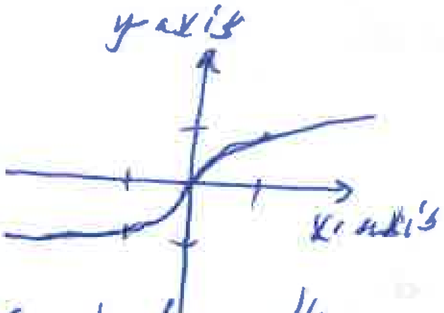
Graph of $y = 3x^2$



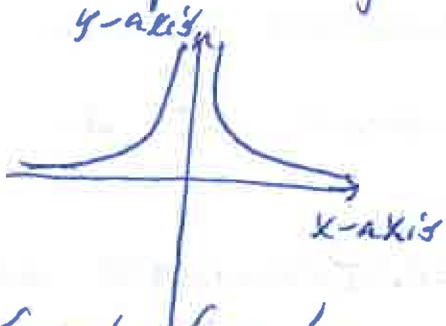
Graph of $6x$

g is increasing on $\mathbb{R}$
g has a stationary point at $x=0$.
g is concave down on $\mathbb{R}_{<0}$
g is concave up on $\mathbb{R}_{>0}$
g is has a point of inflection at $x=0$.

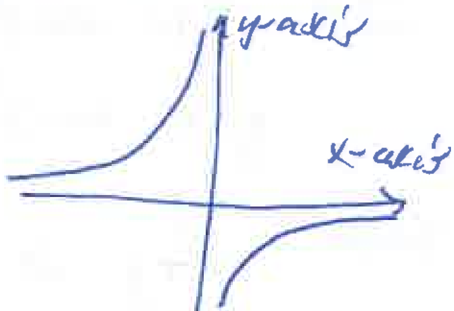
4.23c and 4.25c $h: \mathbb{R} \to \mathbb{R}$ given by $h(x) = x^{1/3}$.



Graph of $y = x^{1/3}$



Graph of $y = \frac{1}{3x^{2/3}}$

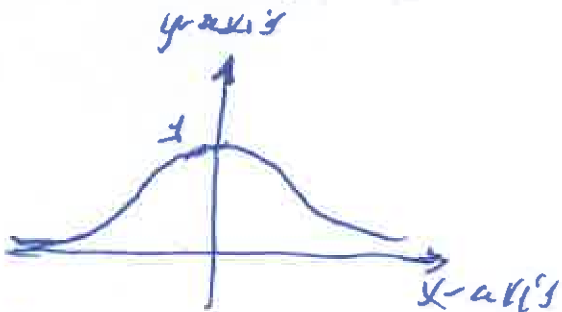


Graph of $y = -\frac{2}{9} \cdot \frac{1}{x^{5/3}}$

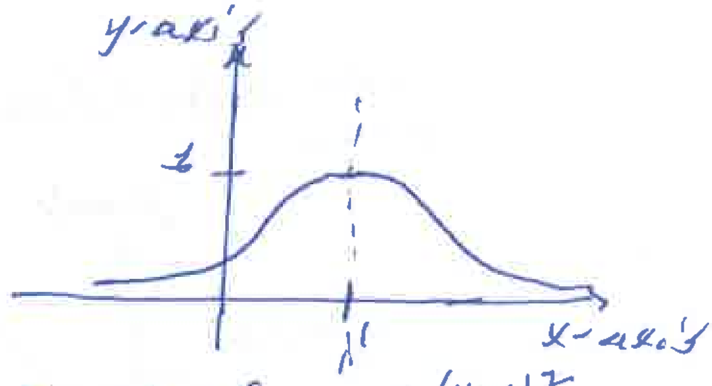
h is increasing on $\mathbb{R}$
h has infinite slope at $x=0$ (a vertical tangent line)
h is concave up on $\mathbb{R}_{<0}$
h is concave down on $\mathbb{R}_{>0}$
h has no maximum and no minimum.

An asymptote of f as $x \to a$ is a line that the graph of $f(x)$ gets closer to as x gets closer to a.

$\underline{4.15}$ $f:\mathbb{R}\to\mathbb{R}$ given by
$$f(x)=e^{-(x-1)^2}$$



The bell curve
$y=e^{-x^2}$



Graph of $y=e^{-(x-1)^2}$.

$$\frac{d\,e^{-x^2}}{dx}=-2xe^{-x^2}$$

If $g(x)=e^{-x^2}$ then

g is increasing on $\mathbb{R}_{<0}$
g is decreasing on $\mathbb{R}_{>0}$
g has a stationary point at $x=0$

f has asymptote $y=0$ as $x\to\infty$ and as $x\to-\infty$.
f is increasing on $\mathbb{R}_{<1}$
f is decreasing on $\mathbb{R}_{>1}$
f has a stationary point at $x=1$

$$\frac{d^2}{dx^2}(e^{-x^2})=(-2x)(-2x)e^{-x^2}+(-2)e^{-x^2}=(4x^2-2)e^{-x^2}$$
$$=4(x^2-\tfrac{1}{2})e^{-x^2}=4\left(x-\tfrac{1}{\sqrt{2}}\right)\left(x+\tfrac{1}{\sqrt{2}}\right).$$

g is concave up on $(-\infty,-\frac{1}{\sqrt2})$ and $(\frac{1}{\sqrt2},\infty)$

g is concave down on $(-\frac{1}{\sqrt2},\frac{1}{\sqrt2})$

g has points of inflection at $x=-\frac{1}{\sqrt2}$ and $x=\frac{1}{\sqrt2}$

f is concave up on $(-\infty,1-\frac{1}{\sqrt2})$ and $(1+\frac{1}{\sqrt2},\infty)$

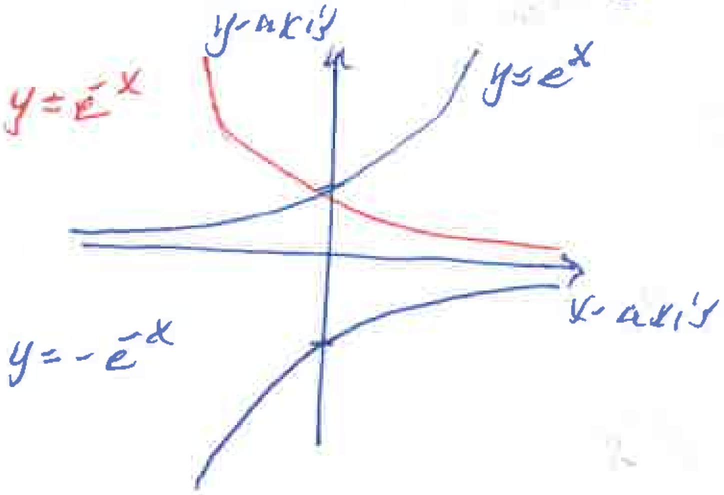
f is concave down on $(1-\frac{1}{\sqrt2},1+\frac{1}{\sqrt2})$

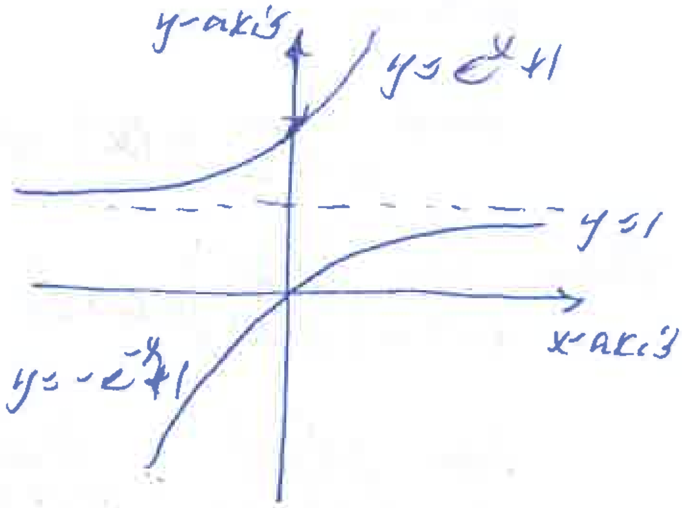
f has points of inflection at $x=1-\frac{1}{\sqrt2}$ and $x=1+\frac{1}{\sqrt2}$.

<u>4.19</u>

$f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^x + 1$

$g: \mathbb{R} \to \mathbb{R}$ given by $g(x) = 1 - e^{-x} = -e^{-x} + 1$





f has asymptote $y = 1$ as $x \to -\infty$

g has asymptote $y = 1$ as $x \to \infty$.