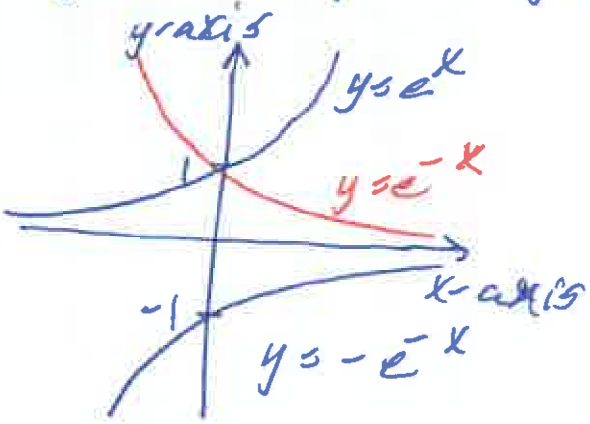
Asymptotes

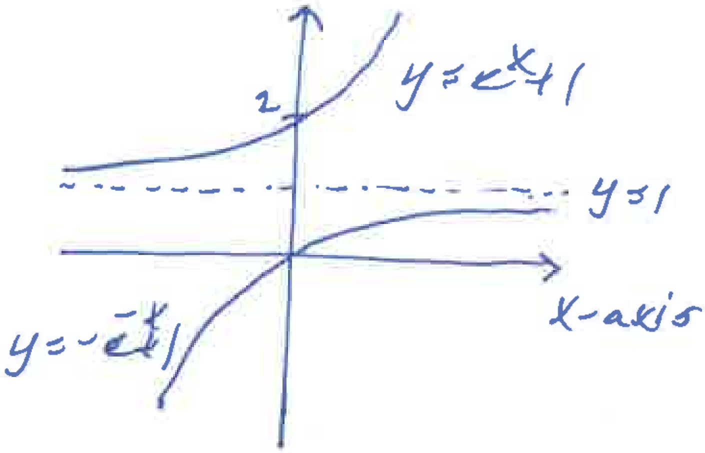
16.04.2025 ①
Calculus Lect 19
A. Ram

4.29

$f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^x + 1$

$g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = 1 - e^{-x} = -e^{-x} + 1$



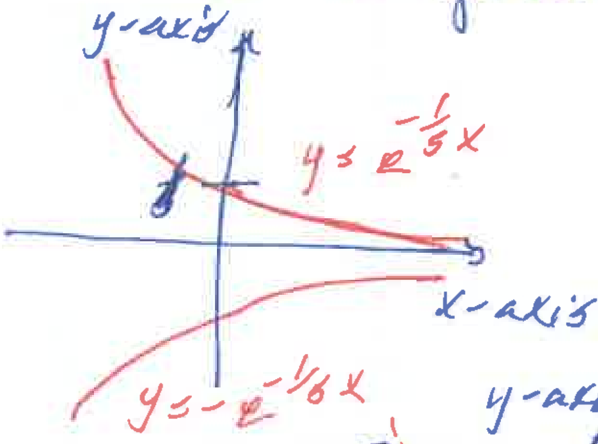


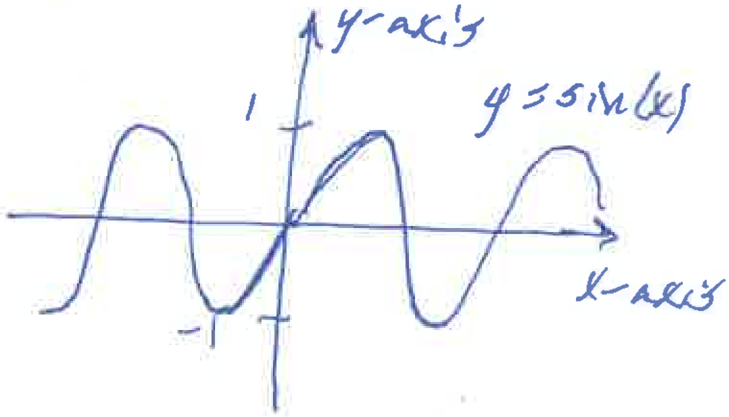
f has asymptote $y = 1$ as $x \to -\infty$

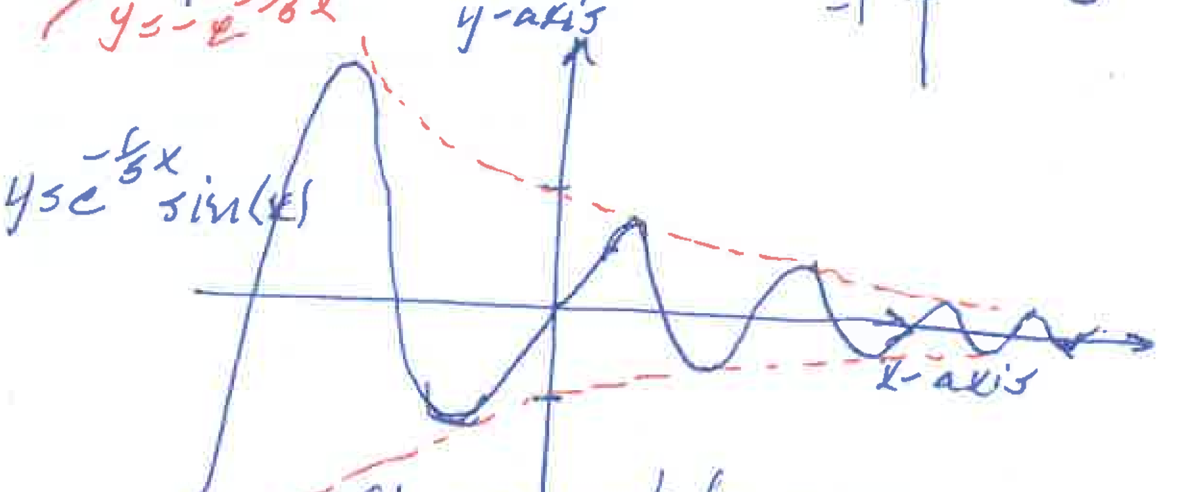
g has asymptote $y = 1$ as $x \to \infty$

4.31

$f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^{-\frac{1}{5}x} \sin(x)$







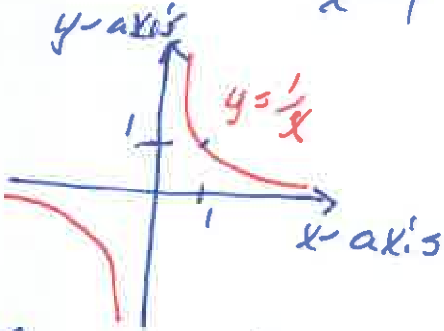
f has asymptote $y = 0$ as $x \to \infty$

4.28 $f : \mathbb{R} \setminus \{-1,1\} \to \mathbb{R}$ given by

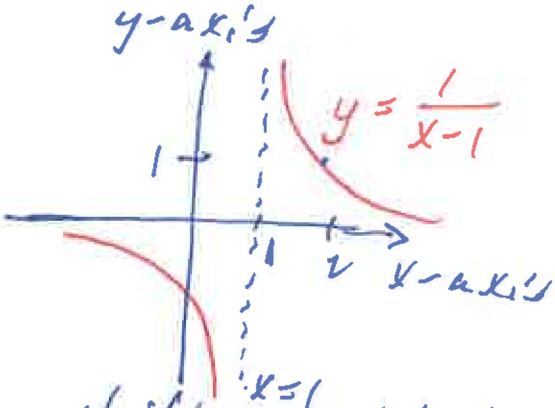
$$f(x) = \frac{x^2+2x+1}{x^2-1} = \frac{(x+1)^2}{(x+1)(x-1)} = \frac{x+1}{x-1} = \frac{x-1+2}{x-1}$$

So

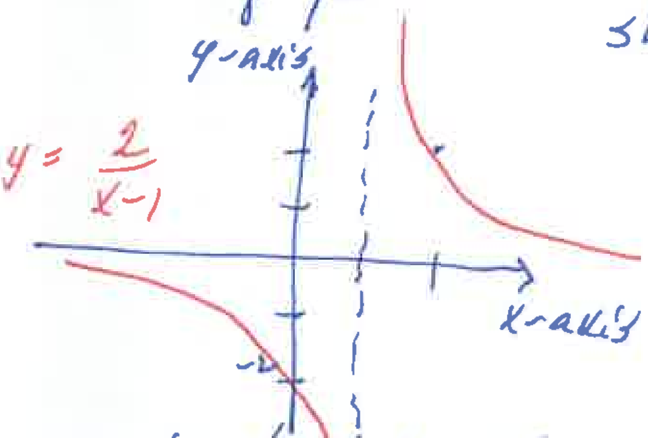
$$f(x) = 1 + \frac{2}{x-1}$$



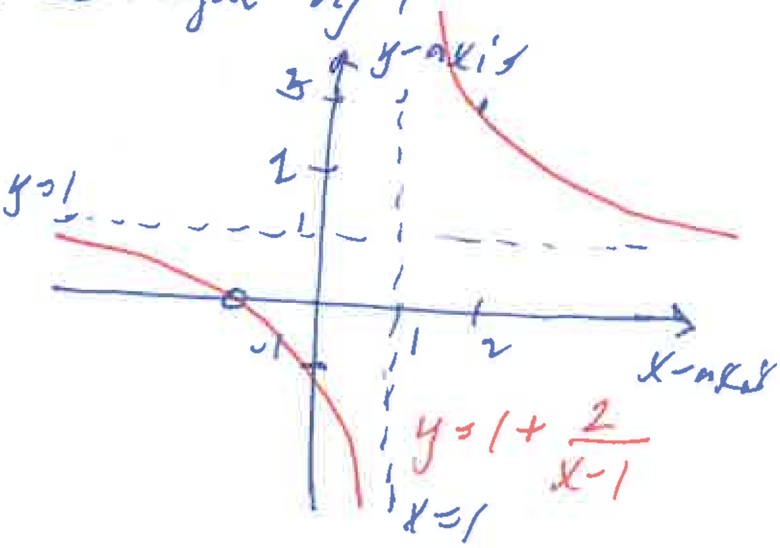
Basic graph



shifted right by 1



y-axis stretched by 2



f has no maximum and no minimum.

f is decreasing for $x \in \mathbb{R} \setminus \{-1,1\}$

f is concave down on $\mathbb{R}_{<1} \setminus \{-1\} = (-\infty,-1) \cup (-1,1)$

f is concave up on $\mathbb{R}_{>1} = (1,\infty)$

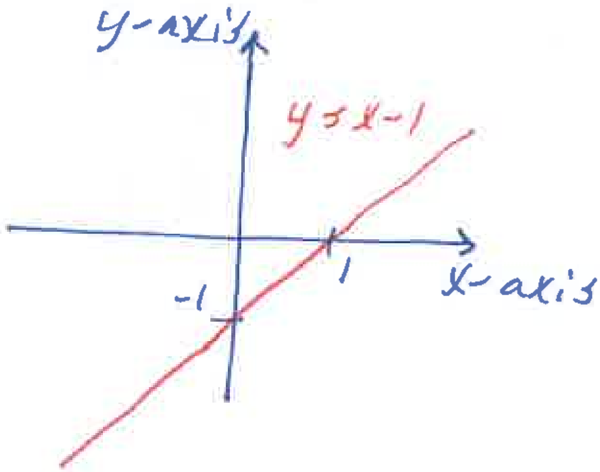
f has no point of inflection.

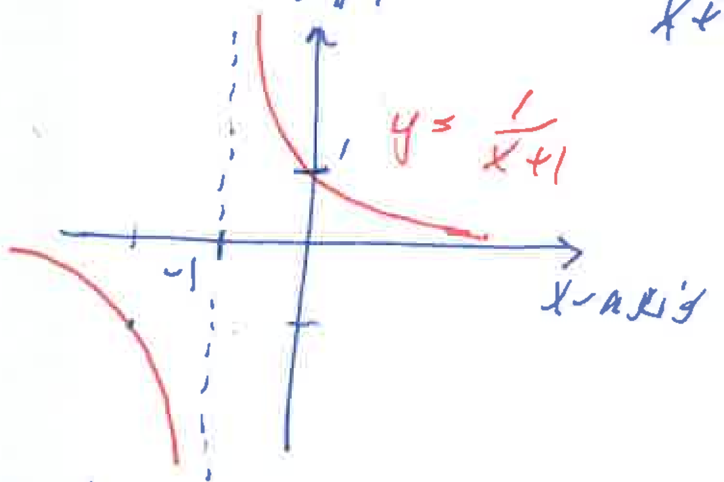
f has asymptote $y=1$ as $x \to \infty$ and asymptote $y=1$ as $x \to -\infty$.

f has asymptote $x=1$ as $x \to 1$.

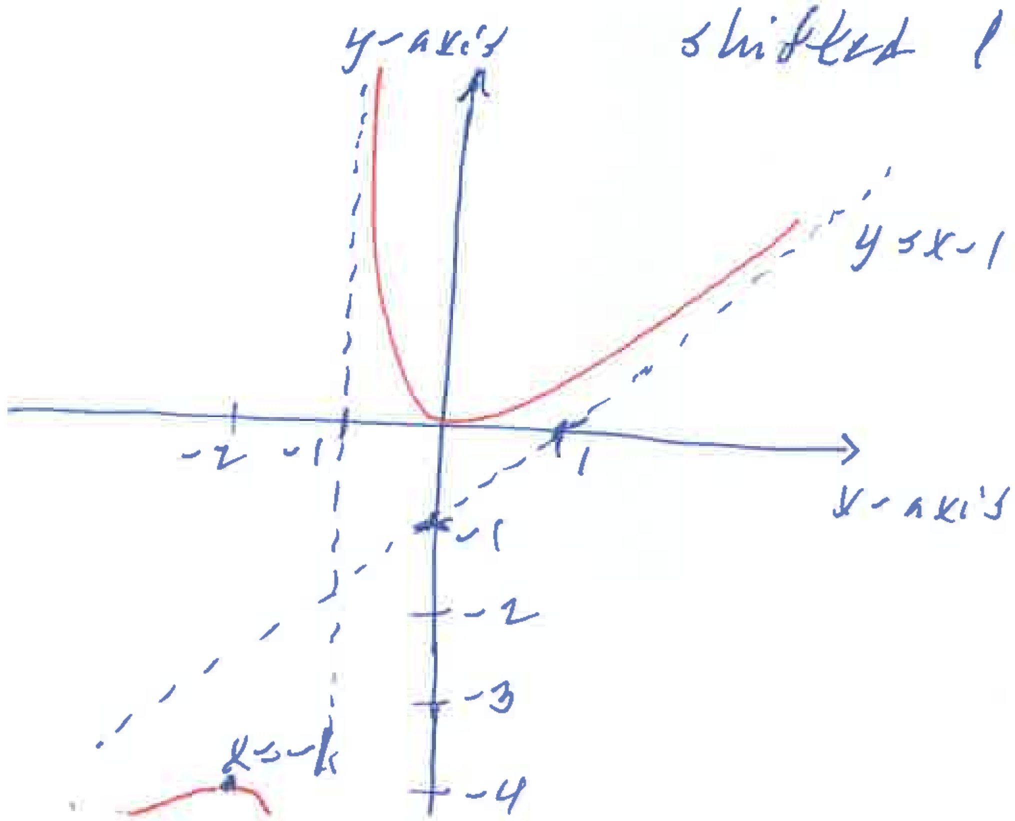
<u>4.32</u> $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \dfrac{x^2}{x+1}$

$$f(x) = \frac{x(x+1)-x}{x+1} = x - \frac{x}{x+1} = x - \frac{x+1-1}{x+1} = x - 1 + \frac{1}{x+1}$$





$y = \frac{1}{x+1}$ is $y = \frac{1}{x}$ but shifted 1 to the left.



Notes:

(a) $f(0) = \frac{0^2}{0+1} = \frac{0}{1} = 0$

$f(-2) = \frac{(-2)^2}{-2+1} = \frac{4}{-1} = -4$

(b) $f'(x) = 1 - 0 - \frac{1}{(x+1)^2} = 1 - \frac{1}{(x+1)^2}$

So $f'(0) = 1 - \frac{1}{1^2} = 1 - 1 = 0$

and $f'(-2) = 1 - \frac{1}{(-2+1)^2} = 1 - \frac{1}{(-1)^2} = 1 - 1 = 0.$

So f has ~~a~~ minimum at $x = 0$

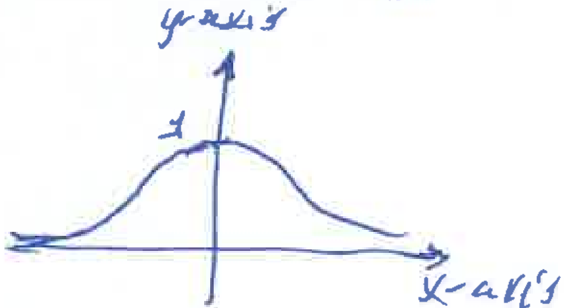
f has a local maximum at $x = -2$.

f has asymptote $y = x - 1$ as $x \to \infty$

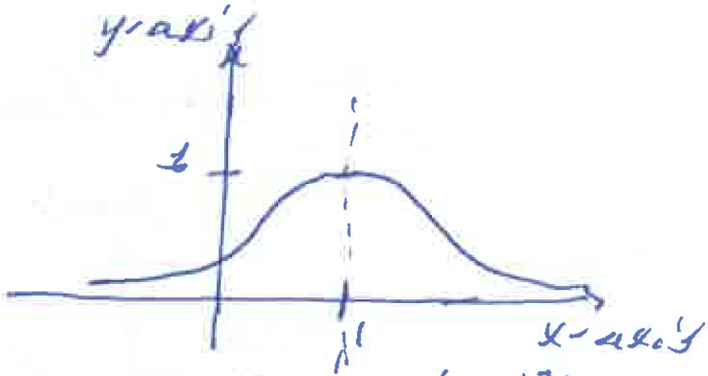
f has asymptote $y = x - 1$ as $x \to -\infty$

f has asymptote $x = -1$ as $x \to -1$.

4.15 $f:\mathbb{R}\to\mathbb{R}$ given by
$f(x)=e^{-(x-1)^2}$



The bell curve
$y=e^{-x^2}$



Graph of $y=e^{-(x-1)^2}$.

$$\frac{d\,e^{-x^2}}{dx}=-2xe^{-x^2}$$

If $g(x)=e^{-x^2}$ then

g is increasing on $\mathbb{R}_{<0}$
g is decreasing on $\mathbb{R}_{>0}$
g has a stationary point at $x=0$

f has asymptote $y=0$ as $x\to\infty$ and as $x\to-\infty$.

f is increasing on $\mathbb{R}_{<1}$
f is decreasing on $\mathbb{R}_{>1}$
f has a stationary point at $x=1$

$$\frac{d^2}{dx^2}(e^{-x^2})=(-2x)(-2x)e^{-x^2}+(-2)e^{-x^2}=(4x^2-2)e^{-x^2}$$
$$=4(x^2-\tfrac{1}{2})e^{-x^2}=4(x-\tfrac{1}{\sqrt{2}})(x+\tfrac{1}{\sqrt{2}}).$$

g is concave up on $(-\infty,-\frac{1}{\sqrt{2}})$ and $(\frac{1}{\sqrt{2}},\infty)$

g is concave down on $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

g has points of inflection at $x=\frac{-1}{\sqrt{2}}$ and $x=\frac{1}{\sqrt{2}}$

f is concave up on $(-\infty,1-\frac{1}{\sqrt{2}})$ and $(1+\frac{1}{\sqrt{2}},\infty)$

f is concave down on $(1-\frac{1}{\sqrt{2}},1+\frac{1}{\sqrt{2}})$

f has points of inflection at $x=1-\frac{1}{\sqrt{2}}$ and $x=1+\frac{1}{\sqrt{2}}$.