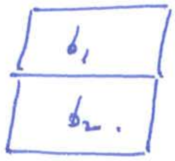
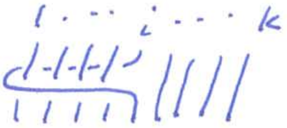
Representation Theory 24.03.2009

B_k is the group of braids on k-strands

$b_1 b_2 =$ 

Let $y^{\varepsilon_i} =$  for $1 \le i \le k$, and $y^{\varepsilon_0} = 1$.

Then
$$y^{\varepsilon_i} y^{\varepsilon_j} = y^{\varepsilon_j} y^{\varepsilon_i} \quad \text{for } 1 \le i, j \le k$$

and $Y = \langle y^{\varepsilon_1}, \ldots, y^{\varepsilon_k} \rangle$ is an abelian subgroup of B_k.

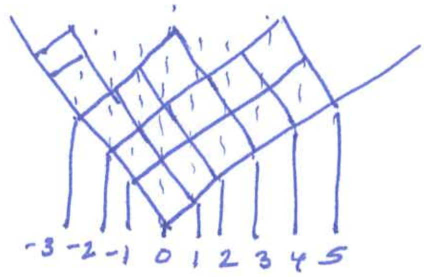
Let $T_{w_0}^2 = y^{\varepsilon_1} \cdots y^{\varepsilon_k} =$  $=$ 

Theorem $Z(B_k) = \langle T_{w_0}^2 \rangle$.

Note that if $z_1 = y^{\varepsilon_1}$, $z_2 = y^{\varepsilon_2} y^{\varepsilon_1}$, $\ldots$, $z_k = y^{\varepsilon_1} \cdots y^{\varepsilon_k}$ then
$$Y = \langle z_1, \ldots, z_k \rangle = \langle Z(B_1), \ldots, Z(B_k) \rangle = \langle y^{\varepsilon_1}, \ldots, y^{\varepsilon_k} \rangle.$$

Note:
If M is a simple B_k-module then
$$T_{w_0}^2 = c \cdot Id_M, \quad \text{as operators on } M.$$

Let λ be a partion with k-boxes.

$\lambda =$ 

A <u>standard tableau T of shape λ</u> is a filling of the boxes of λ with $1, 2, \ldots, k$ such that

and


Let

$$c(T(i)) = (\text{runner number of box } i \text{ in } T)$$

Define

$$H_k^\lambda = \mathrm{span}\left\{ v_T \;\middle|\; \begin{array}{c} T \text{ is a standard tableau of} \\ \text{shape } \lambda \end{array} \right\}$$

with H_k-action

$$y^{\varepsilon_i} v_T = q^{c(T(i))} v_T$$

$$T_i v_T = \frac{q - q^{-1}}{1 - q^{c(T(i)) - c(T(i+1))}} v_T + \left(q^{-1} + \frac{q - q^{-1}}{1 - q^{c(T(i)) - c(T(i+1))}} \right) v_{s_i T}$$

where

$s_i T = T$ except with box i and box $i+1$ switched

$v_{s_i T} = 0$ if $s_i T$ is not standard.

<u>Theorem</u> (a) H_k^{λ} is an H_k-module

(b) H_k^{λ} is a simple H_k-module

(c) The H_k^{λ} are all the simple H_k-modules.

(d) $$\mathrm{Res}^{H_k}_{H_{k-1}}(H_k^{\lambda}) = \bigoplus_{\lambda/\mu = \square} H_{k-1}^{\mu}$$

<u>Example</u> H_5  has basis

$v_{\substack{5\\3\,4\\1\,2}}$, $v_{\substack{4\\3\,5\\1\,2}}$, $v_{\substack{5\\2\,4\\1\,3}}$, $v_{\substack{4\\2\,5\\1\,3}}$, $v_{\substack{3\\2\,5\\1\,4}}$ and

$$T_2 v_{\substack{4\\3\,5\\1\,2}} = \frac{q-q^{-1}}{1-q^{2(1-(-1))}} v_{\substack{4\\3\,5\\1\,2}} + \left(q^{-1} + \frac{q-q^{-1}}{1-q^{2(1-(1-1))}}\right) v_{\substack{4\\2\,5\\1\,3}}$$

$$T_2 v_{\substack{3\\2\,5\\1\,4}} = \frac{q-q^{-1}}{1-q^{2(-1-(-2))}} v_{\substack{3\\2\,5\\1\,4}} + \left(q^{-1} + \frac{q-q^{-1}}{1-q^{2(\cancel{2}1-(-2))}}\right) v_{\substack{2\\3\,5\\1\,4}}$$

$$= -q^{-1} v_{\substack{3\\2\,5\\1\,4}}$$

Recall:

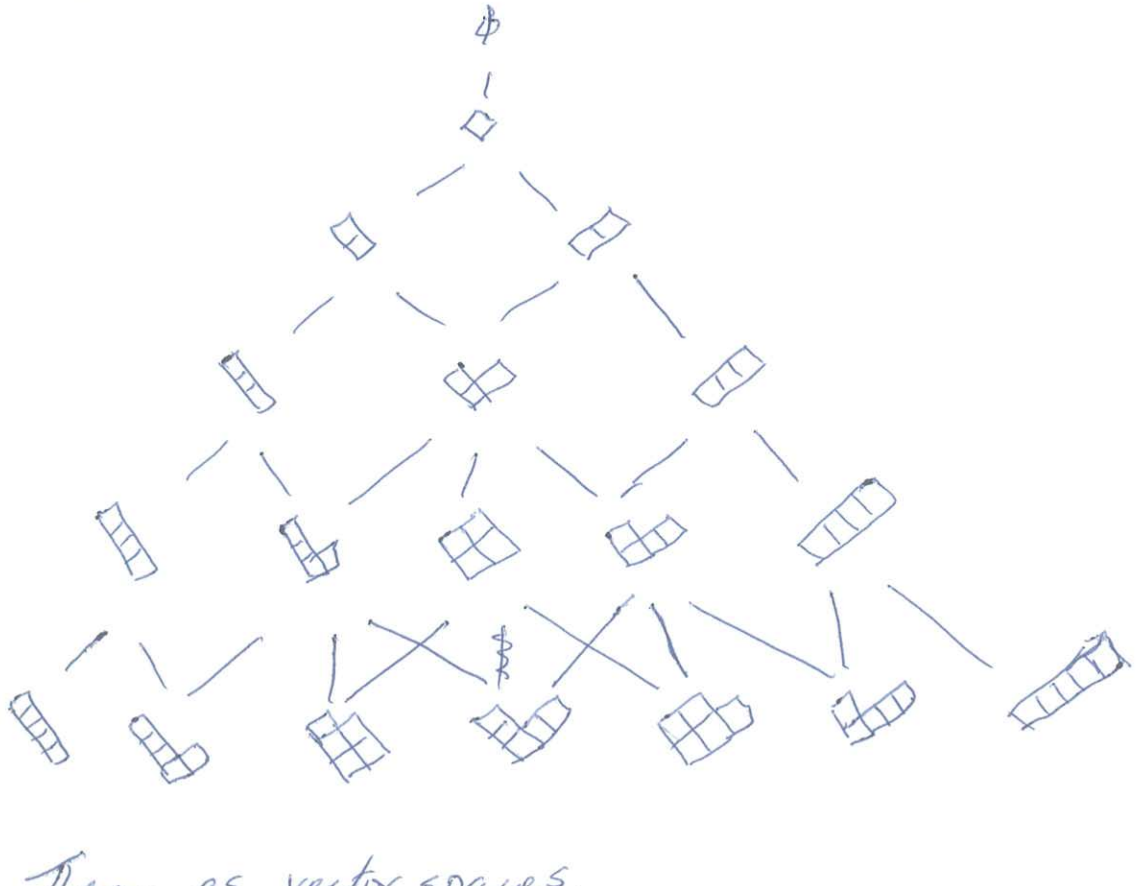
$$\mathbb{C}B_k \twoheadrightarrow H_k \twoheadrightarrow TL_k$$
$$T_i \longmapsto T_i$$
$$e_i \longmapsto e_i$$

where $e_i = q - T_i$.

and that the Bratelli diagram for

$$H_1 \subseteq H_2 \subseteq H_3 \subseteq \cdots$$

is



Then, as vector spaces,

$$\mathrm{Res}^{H_5}_{H_4}(H_5) = H_4 \oplus H_4 = H_3 \oplus H_3 \oplus H_3$$

$$= H_2 \oplus H_2 \oplus H_2 \oplus H_2 \oplus H_2$$

so that $\dim(H$ $) = 5$