Problem Sheet -- Rep Thy
620-619 Semester I 2010

Arun Ram
Department of Mathematics and Statistics
University of Melbourne
Parkville, VIC 3010 Australia
aram@unimelb.edu.au
and

Department of Mathematics
University of Wisconsin, Madison
Madison, WI 53706 USA
ram@math.wisc.edu

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1. Algebras

(1) Define *algebra* and *Lie algebra*.

(2) Define *group algebra* and *enveloping algebra*.

(3) Define *basis* and *multiplication table* of an algebra.

(4) Define *presentation* of an algebra (by generators and relations).

(5) Write (with proof) a basis and the multiplication rule for the Temperley-Lieb algebras TL_1, TL_2, TL_3, TL_4. To do this, you will first have to define these algebras.

(6) Write (with proof) a basis and the multiplication rule for the matrix algebras $M_1(\mathbb{C})$, $M_2(\mathbb{C})$, $M_3(\mathbb{C})$, $M_4(\mathbb{C})$. To do this, you will first have to define these algebras.

(7) Write (with proof) a basis and the multiplication rule for the group algebras of the symmetric groups $\mathbb{C}S_1$, $\mathbb{C}S_2$, $\mathbb{C}S_3$, $\mathbb{C}S_4$. To do this, you will first have to define these algebras.

(8) Write (with proof) a basis and the multiplication rule for the Brauer algebras W_1, W_2, W_3, W_4. To do this, you will first have to define these algebras.

(9) Write (with proof) a basis and the multiplication rule for the Iwahori-Hecke algebras H_1, H_2, H_3, H_4 (of finite type A). To do this, you will first have to define these algebras.

(10) Write (with proof) a basis and the multiplication rule for the tensor algebras. To do this, you will first have to define these algebras.

(11) Write (with proof) a basis and the multiplication rule for the symmetric algebras. To do this, you will first have to define these algebras.

(12) Write (with proof) a basis and the multiplication rule for the exterior algebras. To do this, you will first have to define these algebras.

(13) Write (with proof) a basis and the multiplication rule for the group algebras of the cyclic groups. To do this, you will first have to define these algebras.

(14) Write (with proof) a basis and the multiplication rule for the group algebras of the dihedral groups. To do this, you will first have to define these algebras.

(15) Write (with proof) a basis and the multiplication rule for the group algebras of the alternating groups. To do this, you will first have to define these algebras.

(16) Write (with proof) a basis and the multiplication rule for the group algebras of the tetrahedral group. To do this, you will first have to define these algebras.

(17) Write (with proof) a basis and the multiplication rule for the group algebras of the octahedral group. To do this, you will first have to define these algebras.

(18) Write (with proof) a basis and the multiplication rule for the group algebras of the icosahedral group. To do this, you will first have to define these algebras.

(19) Write (with proof) a basis and the multiplication rule for the group algebras of finite abelian groups. To do this, you will first have to define these algebras.

(20) Write (with proof) a basis and the multiplication rule for the group algebras of $GL_1(\mathbb{C})$, $GL_2(\mathbb{C})$, and $GL_3(\mathbb{C})$. To do this, you will first have to define these algebras.

(21) Write (with proof) a basis and the multiplication rule for the group algebras of $GL_1(\mathbb{Z})$, $GL_2(\mathbb{Z})$, and $GL_3(\mathbb{Z})$. To do this, you will first have to define these algebras.

(22) Write (with proof) a basis and the multiplication rule for the group algebras of $SL_1(\mathbb{C})$, $SL_2(\mathbb{C})$, and $SL_3(\mathbb{C})$. To do this, you will first have to define these algebras.

(23) Write (with proof) a basis and the multiplication rule for the group algebras of $SL_1(\mathbb{Z})$, $SL_2(\mathbb{Z})$, and $SL_3(\mathbb{Z})$. To do this, you will first have to define these algebras.

(24) Write (with proof) a basis and the multiplication rule for the enveloping algebras of $\mathfrak{gl}_1(\mathbb{C})$, $\mathfrak{gl}_2(\mathbb{C})$, and $\mathfrak{gl}_3(\mathbb{C})$. To do this, you will first have to define these algebras.

(25) Write (with proof) a basis and the multiplication rule for the enveloping algebras of $\mathfrak{sl}_1(\mathbb{C})$, $\mathfrak{sl}_2(\mathbb{C})$, and $\mathfrak{sl}_3(\mathbb{C})$. To do this, you will first have to define these algebras.

(26) Provide and prove a presentation of the Temperley-Lieb algebras.

(27) Provide and prove a presentation of the group algebras of the symmetric groups.

(28) Provide and prove a presentation of the Brauer algebras.

(29) Provide and prove a presentation of the Iwahori-Hecke algebras (of finite type A).

(30) Provide and prove a presentation of the tensor algebras.

(31) Provide and prove a presentation of the symmetric algebras.

(32) Provide and prove a presentation of the exterior algebras.

(33) Provide and prove a presentation of the Weyl algebras.

(34) Provide and prove a presentation of the polynomial algebras.

(35) Provide and prove a presentation of the algebra of continuous functions.

(36) Provide and prove a presentation of the algebra of differentiable functions.

(37) Provide and prove a presentation of the algebra of C^r functions.

(38) Provide and prove a presentation of the algebra of holomorphic functions.

(39) Provide and prove a presentation of the algebra of meromorphic functions.

(40) Provide and prove a presentation of the algebra of regular functions.

(41) Provide and prove a presentation of the algebra of complex numbers.

(42) Provide and prove a presentation of the algebra of quaternions.

(43) Provide and prove a presentation of the octonions.

(44) Provide and prove a presentation of the Clifford algebras.

(45) Provide and prove a presentation of the group algebras of the cyclic groups.

(46) Provide and prove a presentation of the group algebras of the dihedral groups.

(47) Provide and prove a presentation of the group algebras of the alternating groups.

(48) Provide and prove a presentation of the group algebras of the tetrahedral group.

(49) Provide and prove a presentation of the group algebras of the octahedral group.

(50) Provide and prove a presentation of the group algebras of the icosahedral group.

(51) Provide and prove a presentation of the group algebras of finite abelian groups.

(52) Provide and prove a presentation of the group algebras of $\mathrm{GL}_1(\mathbb{C})$, $\mathrm{GL}_2(\mathbb{C})$, and $\mathrm{GL}_3(\mathbb{C})$.

(53) Provide and prove a presentation of the group algebras of $\mathrm{GL}_1(\mathbb{Z})$, $\mathrm{GL}_2(\mathbb{Z})$, and $\mathrm{GL}_3(\mathbb{Z})$.

(54) Provide and prove a presentation of the group algebras of $\mathrm{SL}_1(\mathbb{C})$, $\mathrm{SL}_2(\mathbb{C})$, and $\mathrm{SL}_3(\mathbb{C})$.

(55) Provide and prove a presentation of the group algebras of $\mathrm{SL}_1(\mathbb{Z})$, $\mathrm{SL}_2(\mathbb{Z})$, and $\mathrm{SL}_3(\mathbb{Z})$.

(56) Provide and prove a presentation of the enveloping algebras of $\mathfrak{gl}_1(\mathbb{C})$, $\mathfrak{gl}_2(\mathbb{C})$, and $\mathfrak{gl}_3(\mathbb{C})$.

(57) Provide and prove a presentation of the enveloping algebras of $\mathfrak{sl}_1(\mathbb{C})$, $\mathfrak{sl}_2(\mathbb{C})$, and $\mathfrak{sl}_3($

$\mathbb{C}$).

2. Representations

(1) Define *representation*.

(2) Define *module*.

(3) Define *simple module*.

(4) Define *direct sum* and *indecomposable module*.

(5) Show that the kernel of a homomorphism is a module.

(6) Show that the image of a homomorphism is a module.

(7) Show that submodules of the regular representation are left ideals.

(8) Classify and construct all left ideals of the Temperley-Lieb algebras TL_1, TL_2, TL_3, TL_4. Do the same for all the algebras in the previous section.

3. Categories

(1) Define *category*.

(2) Define *functor* and give some interesting examples of functors.

(3) Define *abelian category*, and *linear category* and *2-category*.

(4) Define *kernel* and *cokernel* and *image*.

(5) Define *exact sequence* and *short exact sequence* and *complex* and *acyclic complex*.

4. Groups

(1) Define *group*.

(2) Define *presentation* of a group.

5. References

[Ra] A. Ram, *Notes*,