
Homework Due 7 April 2009

1. Classify and construct the finite dimensional simple modules for cyclic groups.

2. Classify and construct the finite dimensional simple modules for dihedral groups.

3. Classify and construct the finite dimensional simple modules for $U_q\mathfrak{sl}_2$, where $U_q\mathfrak{sl}_2$ is the algebra generated by $E, F, K^{\pm 1}$, with relations
$$KEK^{-1} = q^2E, \quad KFK^{-1} = q^{-2}F, \quad \text{and} \quad EF - FE = \frac{K - K^{-1}}{q - q^{-1}}.$$

4. Define the symmetric group (via permutations).

5. Show that S_k is generated by $s_1, \ldots, s_{k-1}$ with relations
$$s_i^2 = 1, \qquad s_is_{i+1}s_i = s_{i+1}s_is_{i+1}, \qquad \text{and} \qquad s_is_j = s_js_i \text{ for } j \neq i, i \pm 1.$$

6. In the group algebra of the symmetric group $\mathbb{C}S_k$ define
$$m_j = s_{1j} + s_{2j} + \cdots + s_{j-1,j}\,,$$
where s_{ij} is the transposition that switches i and j. Let $m_1 = 0$.

   a. Show that $m_1 + \cdots + m_k$ is an element of the center of $\mathbb{C}S_k$.

   b. Show that $m_im_j = m_jm_i$ for all $1 \leq i, j \leq k$.