620-619 Representation Theory
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2010 Semester I

University of Melbourne
Mathematics Department

Homework Due 31 May 2010

1. Let $\mathbb{C}S_k$ be the group algebra of the symmetric group. Construct explicitly some modules for $\mathbb{C}S_k$ which have a basis of eigenvectors for the m_i. Do this by describing, explicitly, the action of the s_i and the m_i on the basis vectors.
   a. Be sure to prove that the modules you construct are S_k-modules (by showing that the formulas for the action satisfy the necessary relations).
   b. Show that the modules you have constructed are irreducible.
   c. Show that the modules you constructed are pairwise nonisomorphic.
   d. Show that you have constructed all the irreducible S_k-modules.

2. Use the modules constructed in Problem 1 (or find an alternative method) to determine (with proof) the Bratelli diagram for the tower of algebras

$$\mathbb{C}S_1 \subseteq \mathbb{C}S_2 \subseteq \mathbb{C}S_3 \subseteq \cdots .$$

3. Explicitly verify the Weyl character formula for the $\mathfrak{sl}_3$-crystal $B(\rho)$.

4. Explicitly decompose the $\mathfrak{sl}_3$-crystal $B(\rho) \otimes B(\rho)$.

5. Decompose the adjont representation of SO_5 as an $SU_3 \times SU_2 \times U_1$-module.