

Representation Theory: Tuesday Branching rules

①

There are equivalences of categories:

$$\left\{ \begin{array}{l} \text{complex reductive} \\ \text{algebraic groups} \end{array} \right\} \leftrightarrow \left\{ \begin{array}{l} \text{compact} \\ \text{Lie groups} \end{array} \right\} \leftrightarrow \left\{ \begin{array}{l} \mathbb{Z}\text{-reflection} \\ \text{groups} \end{array} \right\}$$

$$GL_n(\mathbb{C}) \longmapsto U_n(\mathbb{C}) \longmapsto (S_n, \mathbb{Z}^n)$$

$$G \longmapsto K, \text{ the maximal compact subgroup} \longmapsto (W_0, \check{\lambda}^*) \text{ with}$$

$$W_0 = N(T)/T, \check{\lambda}^* = \text{Hom}(T, \mathbb{C}^\times)$$

Hence there are equivalences:

$$\{ G\text{-modules} \} \leftrightarrow \{ K\text{-modules} \} \leftrightarrow \{ (W_0, \check{\lambda}^*) \text{ crystals} \}$$

$$L(\lambda) \longmapsto L(\lambda) \longmapsto B(\lambda)$$

Suppose $H \subseteq G$ (or $L \subseteq K$) (or $(W_H, \check{\lambda}^*) \subseteq (W_0, \check{\lambda}^*)$)

Let $L(\lambda)$ be a simple G -module:

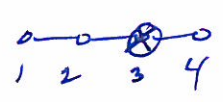
Describe the decomposition of

$$\text{Res}_H^G(L(\lambda)) \text{ into irreducibles.}$$

This should be equivalent to

$$\text{Res}_{(W_H, \check{\lambda}^*)}^{(W_0, \check{\lambda}^*)}(B(\lambda)).$$

Example The adjoint representation of $SU_5(\mathbb{C})$



restricted to $SU_2 \times SU_2 \times U_1$

