Questions for Assignment 3

MAST90017 Representation Theory
Semester II 2015
Lecturer: Arun Ram
to be turned in on 20 August 2015 before 5pm

(1) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let A be a finite dimensional $\mathbb{C}$-algebra. Explain how to determine the indecomposable A-modules from the knowledge of the radical filtration of A.

(2) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let μ_r be a cyclic group of order r and let $G_{r,r,2}$ be the dihedral group of order $2r$. For each of the groups $G = \mu_r$ and $G = G_{r,r,2}$ find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(3) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let p be a prime and let $G = G_p$ be the group of 3×3 upper unitriangular matrices with entries in $\mathbb{F}_p$. For each of these groups find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(4) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let S_n denote the symmetric group. For each of the groups $G = S_2$, $G = S_3$ and $G = S_4$, find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(5) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let A_n denote the alternating group. For each of the groups $G = A_2$, $G = A_3$ and $G = A_4$, find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(6) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) For each of the groups $G = G_{r,1,2}$ with $r \in \mathbb{Z}_{>0}$ find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(7) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) For each of the groups $G = PGL_2(\mathbb{F}_2)$, $G = PGL(\mathbb{F}_3)$ and $G = PGL(\mathbb{F}_4)$ find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(8) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let G be the tetrahedral group. Find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(9) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let G be the octahedral group. Find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(10) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let G be the icosahedral group. Find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(11) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) For each of the finite subgroups G of the group $SU_2(\mathbb{C})$ find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $\mathbb{C}G \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(12) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) $q \in \mathbb{C}^\times$, not a root of unity. Let A be the $\mathbb{C}$-algebra given by generators T_1, T_2 with relations

$$T_i^2 = (q - q^{-1})T_i + 1, \qquad \text{and} \qquad T_1T_2T_1 = T_1T_2T_1.$$

Find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(13) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) $q \in \mathbb{C}^\times$, not a root of unity. Let A be the $\mathbb{C}$-algebra given by generators X_1, T_1 with relations

$$(X_1 - 365)(X_1 - \sqrt{751})(X_1 - 38)(X_1 - \pi) = 0, \qquad T_1^2 = (q - q^{-1})T_1 + 1,$$

$$\text{and} \qquad X_1T_1X_1T_1 = T_1X_1T_1X_1.$$

Find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(14) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let $n = 3642$ and let $TL_k(n)$ denote the Temperley-Lieb algebra spanned by diagrams on k dots without crossings. For $A = TL_2(n)$, $A = TL_3(n)$ and $A = TL_4(n)$, find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(15) (a) Let A be a finite dimensional $\mathbb{C}$-algebra. Show that every indecomposable A-module is simple if and only if there exist positive integers ℓ and $n_1, \ldots, n_\ell$ such that $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.

(b) Let $n = 3642$ and let $B_k(n)$ denote the Brauer algebra spanned by diagrams on k dots (with a product as in the Temperley-Lieb algebra except that crossings of edges are now allowed). For $A = B_2(n)$ and $A = B_3(n)$, find positive integers ℓ and $n_1, \ldots, n_\ell$ and an explicit isomorphism $A \to M_{n_1}(\mathbb{C}) \oplus \cdots \oplus M_{n_\ell}(\mathbb{C})$.