13.05.2026 (1)
Calculus Lect.
A.Ram

Flies

A <u>fly in $\mathbb{R}^2$</u> is a function $r:\mathbb{R}\to\mathbb{R}^2$

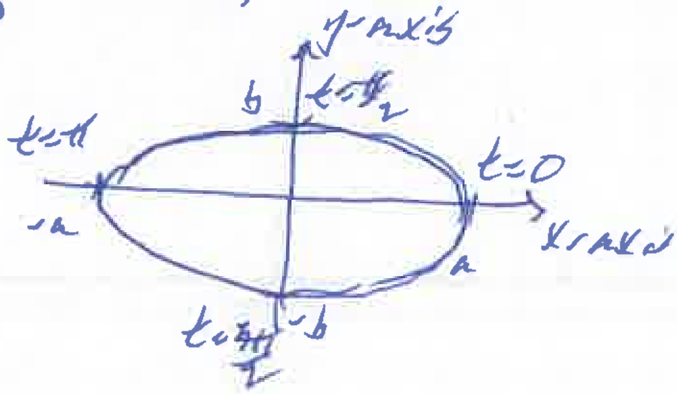
A <u>fly in $\mathbb{R}^3$</u> is a function $r:\mathbb{R}\to\mathbb{R}^3$

Examples of flies in $\mathbb{R}^2$

$$r(t) = x(t)\hat{i} + y(t)\hat{j} = (x(t), y(t))$$

(1) The fly
$$r(t) = a\cos(t)\hat{i} + b\sin(t)\hat{j} \quad \text{with } a,b\in\mathbb{R}_{>0}$$
has $x(t) = a\cos(t)$, $y(t) = b\sin(t)$ so $\frac{x}{a} = \cos(t)$ and $\frac{y}{b} = \sin(t)$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$



(2) The fly $r(t) = \cos(t)\hat{i} + \sin(t)\hat{j}$ is fly (1) with $a=1$ and $b=1$ (a perfect circle).

(3) The fly $r(t) = \cos(2t)\hat{i} + \sin(2t)\hat{j}$ is a perfect circle again except the fly flies twice as fast around the circle.

13.05.2026 (2)
Calculus Lect.
A. Ram

<u>Problem</u> Let $r_1 : \mathbb{R} \to \mathbb{R}^2$ and $r_2 : \mathbb{R} \to \mathbb{R}^2$ be flies given by

$$r_1(t) = (t+1)\hat{i} + (t^2 - 4t)\hat{j} \quad \text{and} \quad r_2(t) = 2t\hat{i} + (6t-9)\hat{j}.$$

(a) Determine the times and points at which the flies collide.

(b) Find the distance between the flies at $t=9$.

(a) If $r_1(t) = r_2(t)$ then $t+1 = 2t$ and
$$t^2 - 4t = 6t - 9.$$

So $1 = t$ and $1^2 - 4 = 6 - 9$.

So $t = 1$ and then $r_1(1) = (1+1)\hat{i} + (1^2 - 4 \cdot 1)\hat{j}$
$$= 2\hat{i} - 3\hat{j} = (2, -3).$$

is where they collide.

(b) The distance between $r_1(9)$ and $r_2(9)$ is

$$\|r_2(9) - r_1(9)\| = \|(2\cdot 9\hat{i} + (6\cdot 9 - 9)\hat{j}) - ((9+1)\hat{i} + (9^2 - 4\cdot 9)\hat{j})\|$$
$$= \|(18\hat{i} + 45\hat{j}) - (10\hat{i} + 45\hat{j})\|$$
$$= \|(18-10)\hat{i} + (45-45)\hat{j}\| = \|8\hat{i}\| = \|(8,0)\|$$
$$= \sqrt{8^2 + 0^2} = 8.$$

13.05.2026 Calculus Lect ③
A.Ram

<u>For $r_1(t)$:</u>

$x = t+1$ and $t = x-1$

$y = t^2 - 4t$

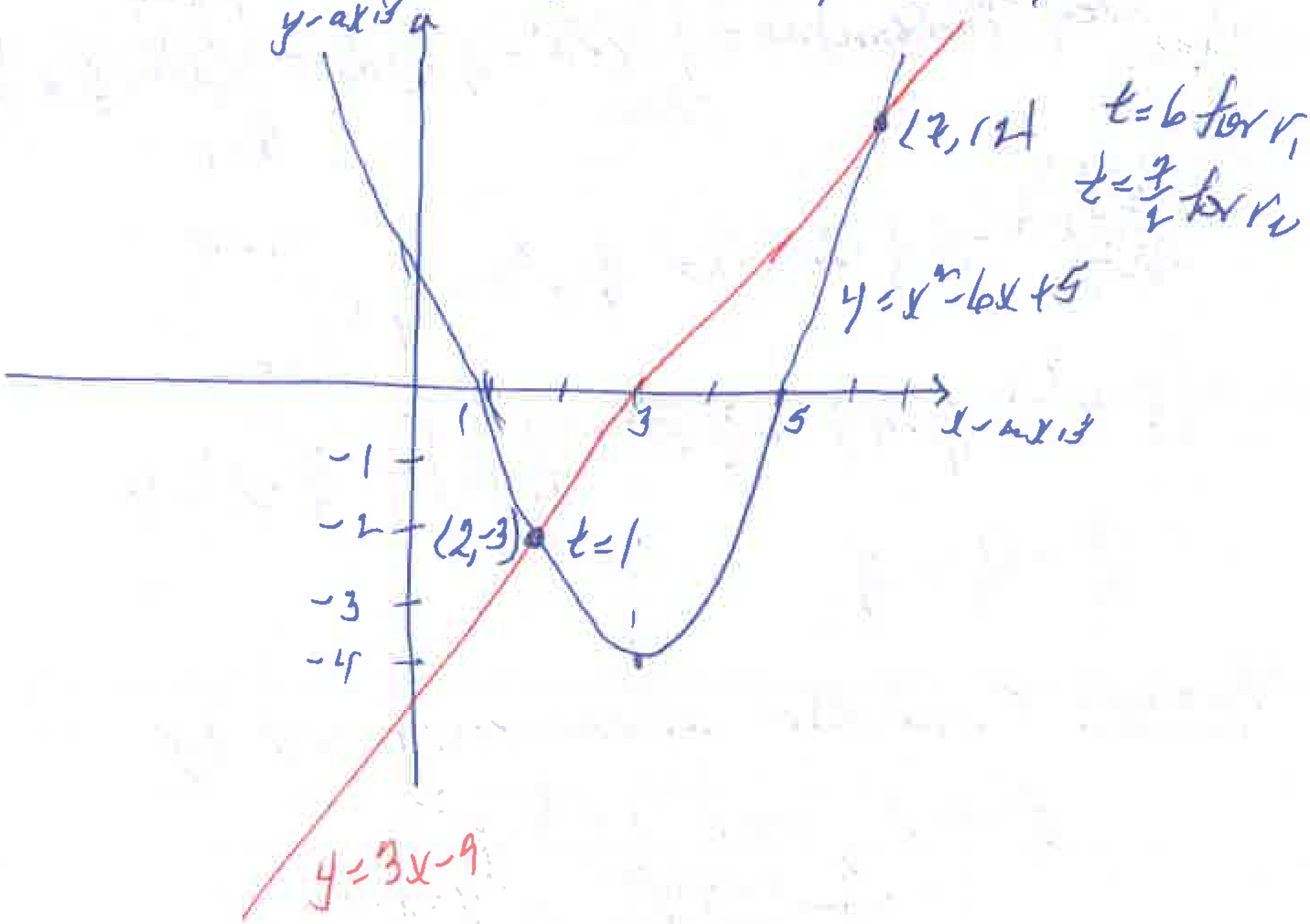
$$y = (x-1)^2 - 4(x-1)$$
$$= (x-1)(x-1-4) = (x-1)(x-5)$$
$$= x^2 - 6x + 5.$$

<u>For $r_2(t)$:</u>

$x = 2t$ and $t = \frac{1}{2}x$

$y = 6t - 9$

$$y = 6 \cdot \tfrac{1}{2}x - 9 = 3x - 9$$



13.05.2026 ④

Calculus Lect
A. Rau

Velocity, acceleration, speed

Position of the fly:

$$\vec{r} = \vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} \quad \text{for } t \in \mathbb{R}$$

Velocity of the fly:

$$\vec{V} = \vec{r}\,'(t) = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

The speed is

$$s = \|r'(t)\| = \|\vec{V}\|$$

The acceleration is

$$\vec{a} = r''(t) = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} = \frac{d\vec{V}}{dt}$$

Problem $r(t) = 2t\,\hat{i} - (3t^3+1)\,\hat{j}$.

$$\vec{V} = r'(t) = 2\hat{i} - 9t^2\hat{j}$$

$$\vec{a} = r''(t) = -18t\,\hat{j}$$

$$s = \|\vec{V}\| = \sqrt{2^2 + (-9t^2)^2} = (4 + 81t^4)^{1/2}.$$

When is speed 0? Never.

When is speed increasing?

When is s^2 increasing? $\quad s^2 = 4 + 81t^4$

When is $\frac{d(s^2)}{dt} > 0$? When $4 \cdot 81 t^3 > 0$