14.06.2026

CalculusLec ①

A. Ran

Flies: When is speed increasing?

Position: $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$.

Velocity $\vec{v} = \vec{r}\,'(t) = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$.

Acceleration: $\vec{a} = \vec{v}\,' = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath}$.

Speed: $s = \|\vec{v}\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$$s^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

$s$ is increasing when $s^2$ is increasing.

$s^2$ is increasing when $\frac{d(s^2)}{dt} > 0$.

$$\frac{d(s^2)}{dt} = 2\frac{dx}{dt}\cdot\frac{d\left(\frac{dx}{dt}\right)}{dt} + 2\frac{dy}{dt}\cdot\frac{d\left(\frac{dy}{dt}\right)}{dt}$$

$$\langle\vec{v},\vec{a}\rangle = \frac{dx}{dt}\cdot\frac{d^2x}{dt^2} + \frac{dy}{dt}\cdot\frac{d^2y}{dt^2}.$$

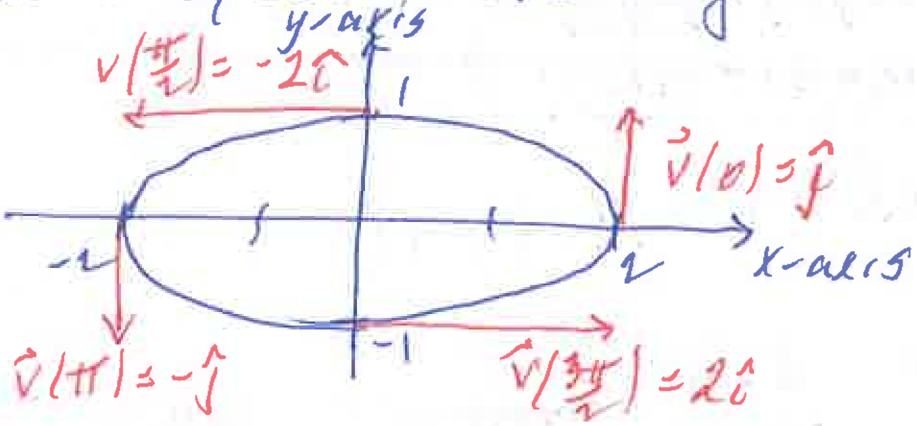
So $\frac{d(s^2)}{dt} = 2\langle\vec{v},\vec{a}\rangle$.

So $s^2$ is increasing when $\langle\vec{v},\vec{a}\rangle > 0$.

14.05.2016 ②
Calculus Lect.
A. Ram

Problem $r(t) = 2\cos(t)\hat{i} + \sin(t)\hat{j}$

When is speed increasing




$\vec{v}(t) = -2\sin(t)\hat{i} + \cos(t)\hat{j}$

$\vec{a}(t) = -2\cos(t)\hat{i} - \sin(t)\hat{j}$

$s = \sqrt{(-2\sin(t))^2 + \cos(t)^2} = \sqrt{4\sin(t)^2 + \cos(t)^2}$

$= \sqrt{3\sin(t)^2 + 1}$

Since $s > 1$ then $s$ is never $0$.

$s^2 = 3\sin(t)^2 + 1$ and $\frac{ds^2}{dt} = 6\sin(t)\cos(t)$
$= 3\sin(2t).$

$\langle \vec{v}, \vec{a} \rangle = 4\sin(t)\cos(t) - \sin(t)\cos(t) = 3\sin(t)\cos(t)$

So $s^{2\prime} = 2\langle \vec{v}, \vec{a} \rangle$.

Speed is increasing when $3\sin(2t) > 0$.

Speed is increasing for $t$ on

$$\bigcup_{k \in \mathbb{Z}} (2k\pi, 2k\pi + \frac{\pi}{2})$$

## Why do we like $\hat{i}, \hat{j}, \hat{k}$?

They are unit vectors

$$\langle \hat{i}, \hat{i} \rangle = \langle (1,0,0), (1,0,0) \rangle = 1+0+0 = 1$$

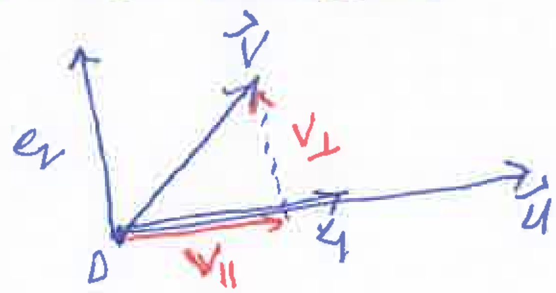
so $\|\hat{i}\| = \sqrt{\langle \hat{i}, \hat{i} \rangle} = \sqrt{1} = 1.$

They are perpendicular

$$\langle \hat{i}, \hat{j} \rangle = \langle (1,0,0), (0,1,0) \rangle = 0\cdot 1 + 1\cdot 0 + 0\cdot 0 = 0.$$

So $\langle \hat{i}, \hat{j} \rangle = 0.$

Let $\vec{u}, \vec{v}$ be vectors




$\vec{v} = \vec{v}_\parallel + \vec{v}_\perp$ Let $e_1 = \dfrac{\vec{u}}{\|\vec{u}\|}$.

$e_1$ is a unit vector parallel to $\vec{u}$.

Let $e_2$ be a unit vector perpendicular to $\vec{u}$

Well $v_\parallel = c_1 e_1$ and $v_\perp = c_2 e_2$. What are $c_1$ and $c_2$?

$$\vec{v} = v_\parallel + v_\perp = c_1 e_1 + c_2 e_2$$

and

$$\begin{aligned}\langle \vec{v}, \tfrac{1}{\|\vec{u}\|}\vec{u} \rangle = \langle \vec{v}, e_1 \rangle &= \langle c_1 e_1 + c_2 e_2, e_1 \rangle \\ &= c_1 \langle e_1, e_1 \rangle + c_2 \langle e_2, e_1 \rangle \\ &= c_1 \cdot 1 + c_2 \cdot 0 = c_1\end{aligned}$$

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Calculus lect (4)
A. Rom

So $e_1 = \frac{1}{\|\vec{u}\|}\langle \vec{v}, \vec{u} \rangle$ and

$$v_{\|} = \frac{1}{\|\vec{u}\|}\langle \vec{v}, \vec{u} \rangle e_1 = \langle \vec{v}, \vec{u} \rangle \frac{1}{\|\vec{u}\|}\frac{1}{\|\vec{u}\|}\vec{u}$$

$$= \frac{\langle \vec{v}, \vec{u} \rangle}{\|\vec{u}\|^2}\vec{u}$$

So $\text{proj}_{\vec{u}}(\vec{v}) = v_{\|} = \frac{\langle \vec{v}, \vec{u} \rangle}{\|\vec{u}\|^2}\vec{u}$.

We used the properties: If $\vec{u}, \vec{w}, \vec{z} \in \mathbb{R}^3$ and $c_1, c_2, c \in \mathbb{R}$ then

(a) $\langle \vec{w}, \vec{u} \rangle = \langle \vec{u}, \vec{w} \rangle$

(b) $\langle c_1 \vec{w} + c_2 \vec{z}, \vec{u} \rangle = c_1 \langle \vec{w}, \vec{u} \rangle + c_2 \langle \vec{z}, \vec{u} \rangle$.

(c) $\langle c\vec{w}, \vec{u} \rangle = c\langle \vec{w}, \vec{u} \rangle = \langle \vec{w}, c\vec{u} \rangle$

(b) should remind you of

$$\frac{d\,(c_1 f + c_2 g)}{dx} = c_1 \frac{df}{dx} + c_2 \frac{dg}{dx}$$

$$\int (c_1 f + c_2 g)\,dx = c_1 \int f\,dx + c_2 \int g\,dx.$$